PSI Center for Accelerator Science and Engineering

Beta function measurement at the SLS 2.0 storage ring

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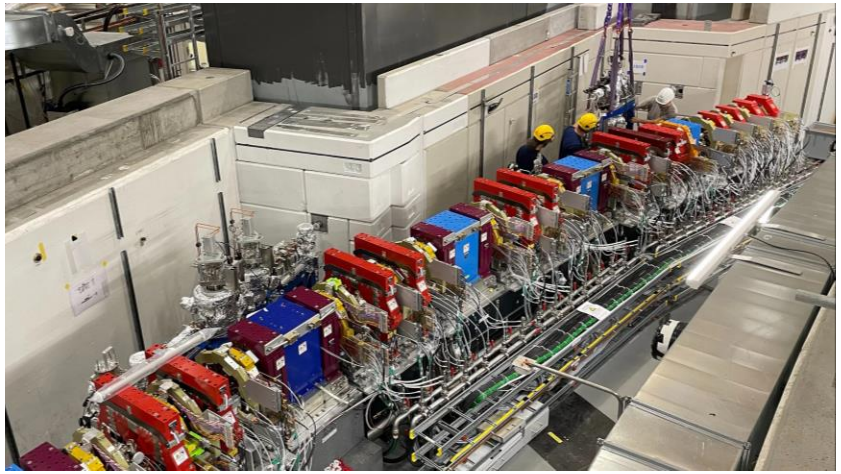
SLS 2.0

Parameters	SLS	SLS 2.0
Lattice type	TBA	7-BA
Number of arcs	12	12
Circumference (m)	288	288
Gross straight length (m)	79.9	83.6
Total bending angle (deg)	374.69	430.08
Working point Q_x/Q_y	20.43/8.74	39.37/15.22
Momentum compaction factor, first/second order (10^{-4})	6.04/36.3	1.05/7.94
Natural chromaticity ξ_x/ξ_y	−67.3/− 21.0	−99.0/− 33.4
Vertical emittance (pm)	≈10	10
Chromaticity in operation	5	1.0–1.5
Energy (GeV)	2.411	2.700
Natural emittance (pm)	5630	158 (135)
Energy spread (10^{-3})	0.88	1.16 (1.04)
Radiation loss per turn (keV)	549	688 (915)
Damping partition $J_x/J_y/J_s$	1.0/1.0/2.0	1.83/1.0/1.17
Damping time $\tau_x/\tau_y/\tau_s$ (ms)	8.65/8.67/4.34	4.14/7.58/6.47
Beam current (mA)	400	400
Maximum rf voltage (MV)	2.6	2.2
Harmonic number	480	480
Number of bunches	390–420	450
Beam lifetime (h)	≈10	≈9

[1]



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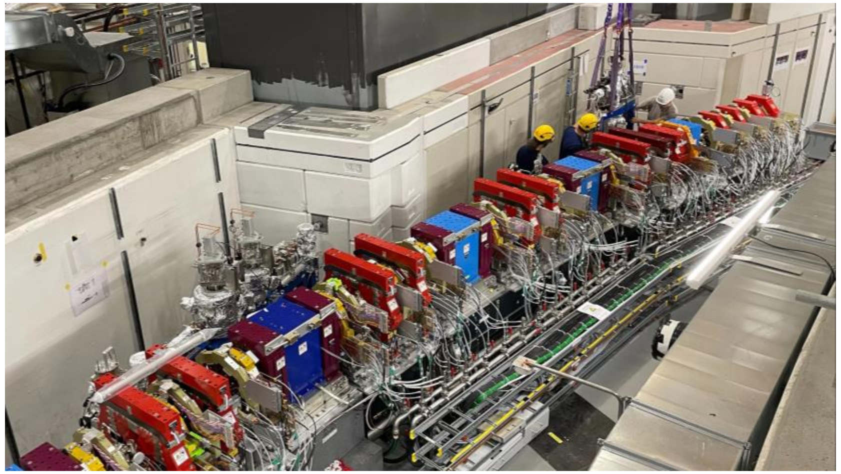
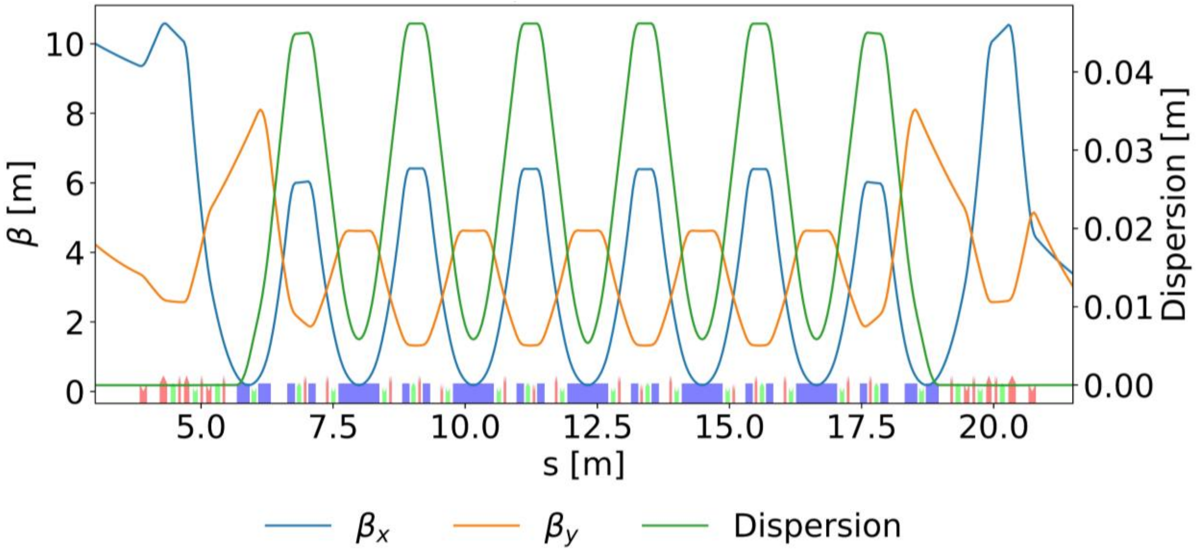




Beta function

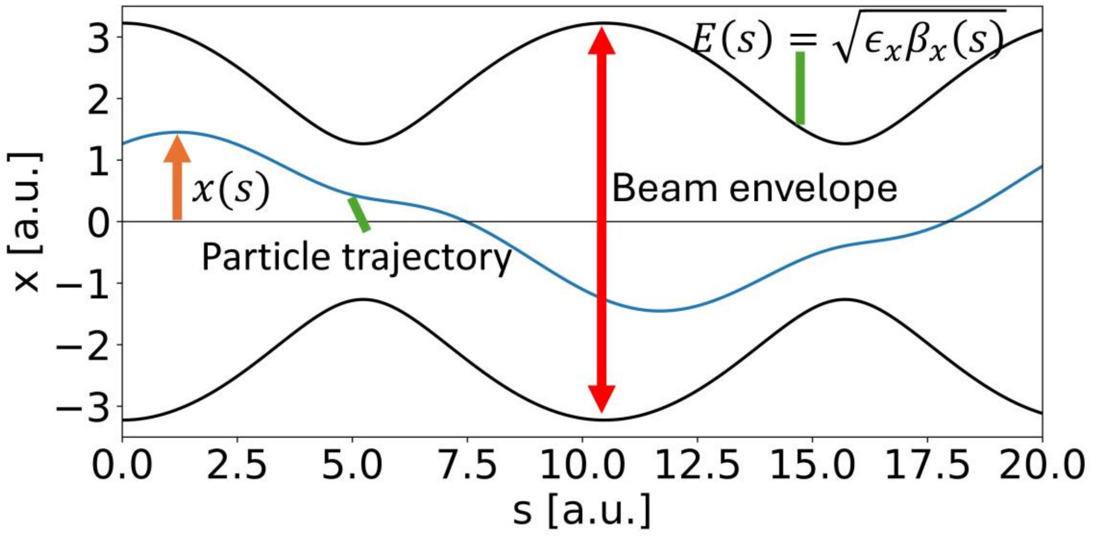
- Trajectory of a particle in the beam (**betatron oscillations**):

$$x(s) = \sqrt{2J_x\beta_x(s)}\cos[\varphi(s) + \vartheta]$$

- **Beam envelope**:

$$E(s) = \pm\sqrt{\epsilon_x\beta_x(s)}, \qquad \epsilon_x = \sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}$$

- ϵ_x the **emittance**
- β_x is the **beta function**, it describes the transverse focusing properties of the storage ring
- If β_x is different than expected, the beam quality is degraded



Quadrupole magnets, tune, and beta functions

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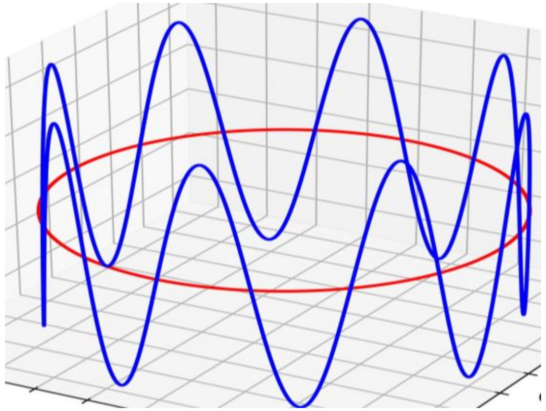
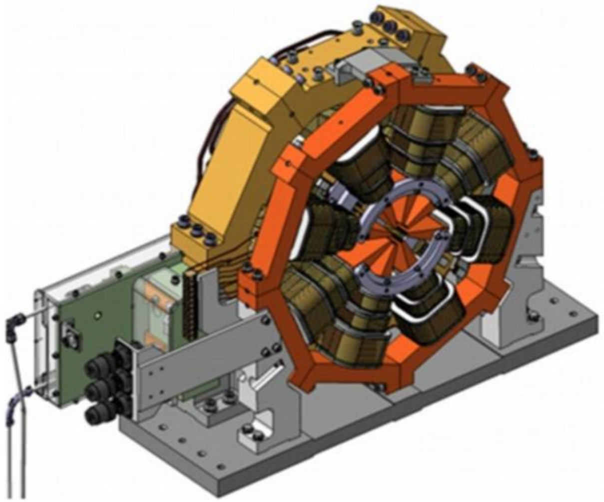
- **Tune** ($Q_{x,y}$): number of betatron oscillations per turn,

$$Q_{x,y} = \frac{1}{2\pi}\oint \frac{ds}{\beta_{x,y}}$$

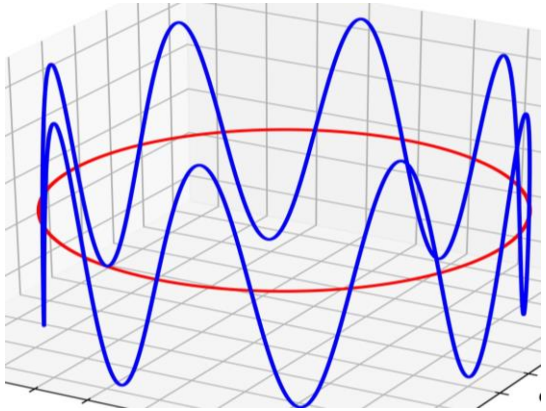
- Quadrupole magnets provide focusing and defocusing to the beam. Changing their strength (ΔK) changes the tune ($\Delta Q_{x,y}$) by,

$$\Delta Q_{x,y} \approx \pm \frac{\beta_{x,y}\Delta K}{4\pi} \longrightarrow \beta_{x,y} \approx \pm 4\pi \frac{\Delta Q_{x,y}}{\Delta K}$$





Quadrupole magnets, tune, and beta functions



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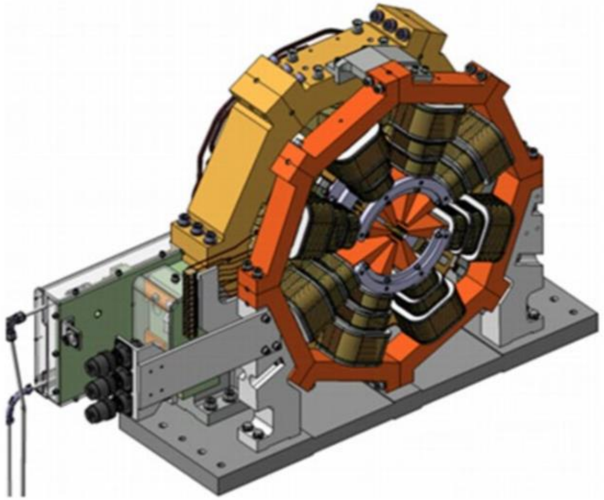
$$Q_{x,y} = \frac{1}{2\pi}\oint \frac{ds}{\beta_{x,y}}$$

- Quadrupole magnets provide focusing and defocusing to the beam. Changing their strength (ΔK) changes the tune ($\Delta Q_{x,y}$). The beta function can be obtained from

$$\beta_{x,y} = \pm\frac{2}{\Delta K}\left\{ \cot(2\pi Q_{x,y})\left[1-\cos(2\pi\Delta Q_{x,y})\right] + \sin(2\pi\Delta Q_{x,y})\right\}$$ [2]

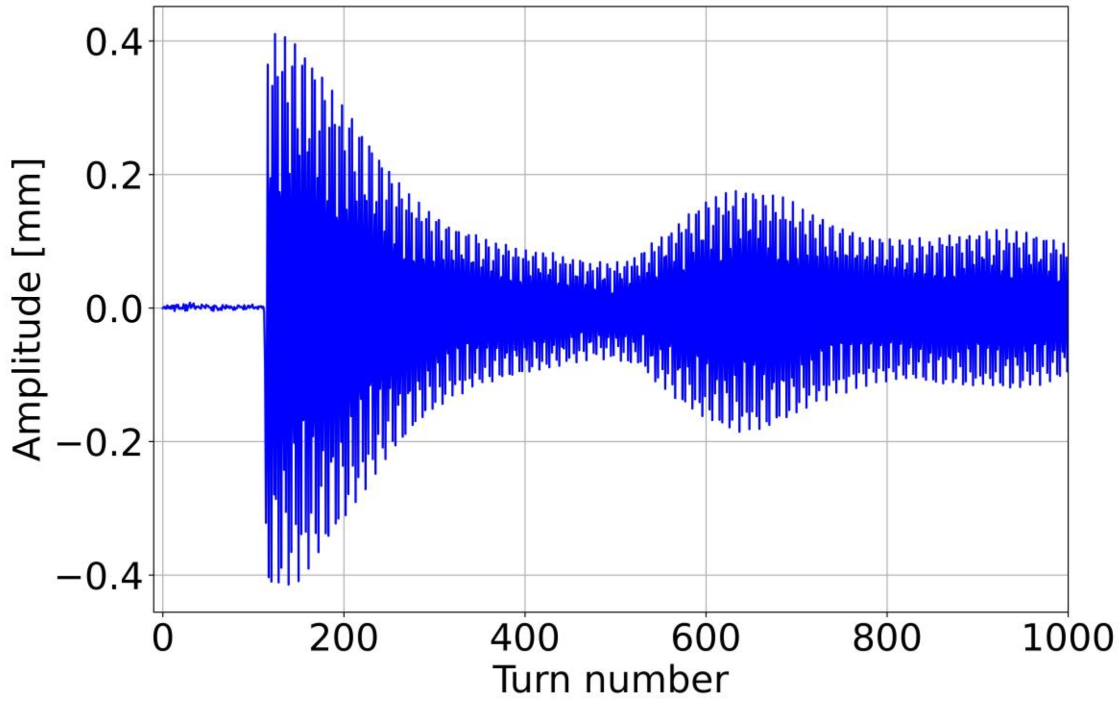


- This method is known as **quadrupole variation** (QV). It is a “direct” measurement

- SLS 2.0 has 264 quadrupoles and 115 beam position monitors (BPMs)

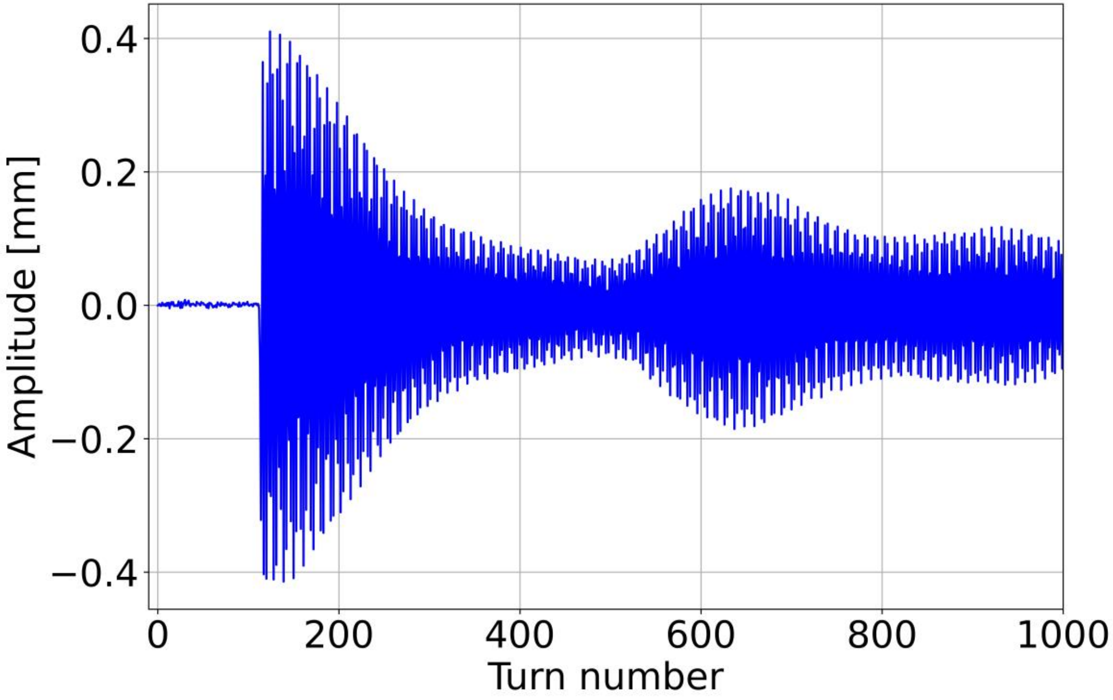
Measuring tune

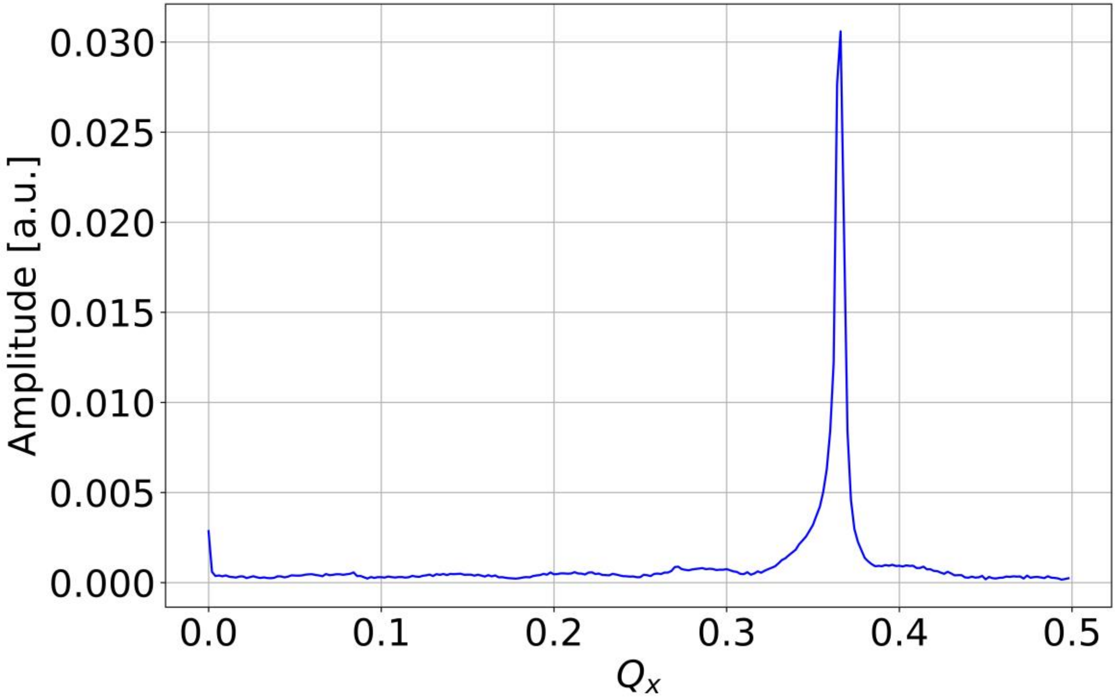
1. Excite the beam
2. Record the centroid coordinates as a function of the number of turns



Measuring tune

1. Excite the beam
2. Record the centroid coordinates as a function of the number of turns
3. Perform a Fourier Transform*



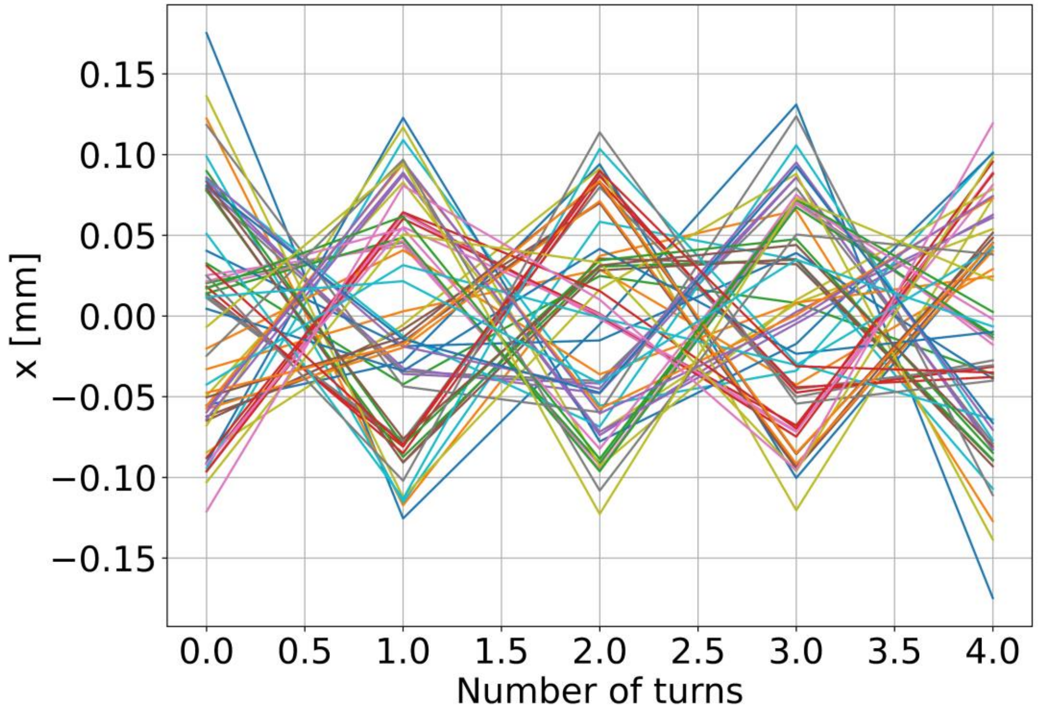


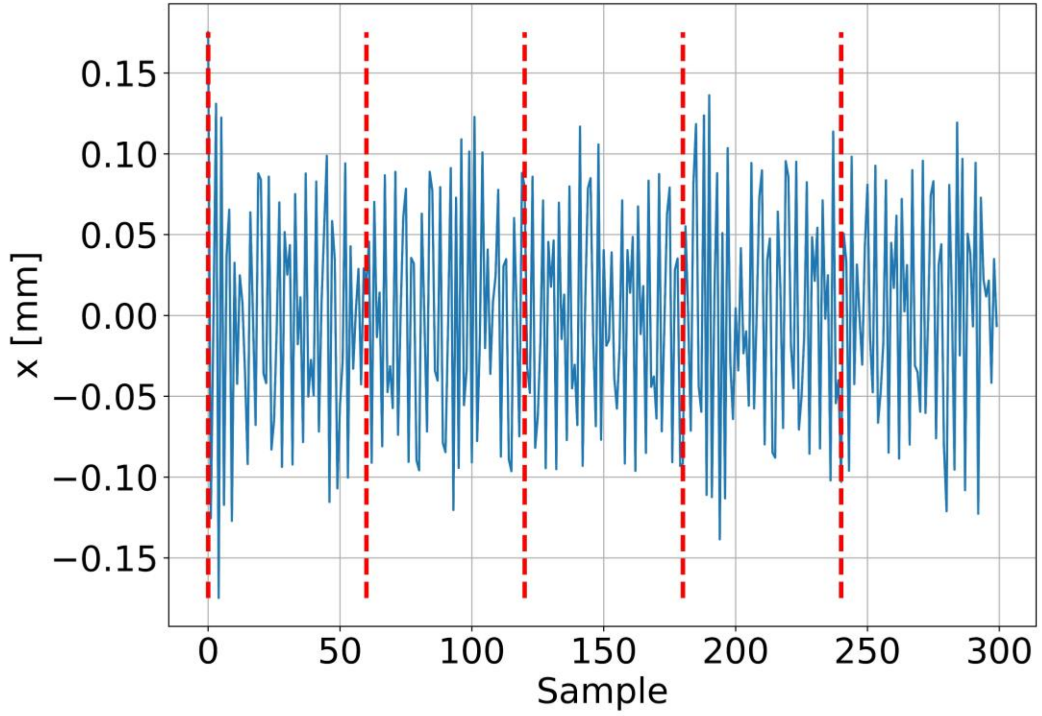
* Tune was determined using the NAFF (Numerical Analysis of Fundamental Frequencies) method.

Mixed BPM Method

[3]

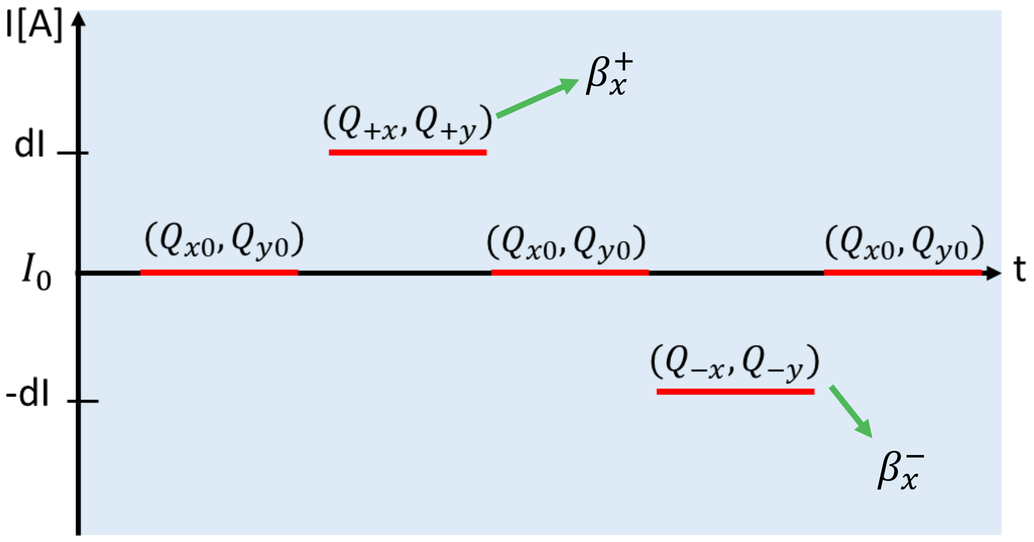
- Use data of M BPMs for N turns with NAFF method



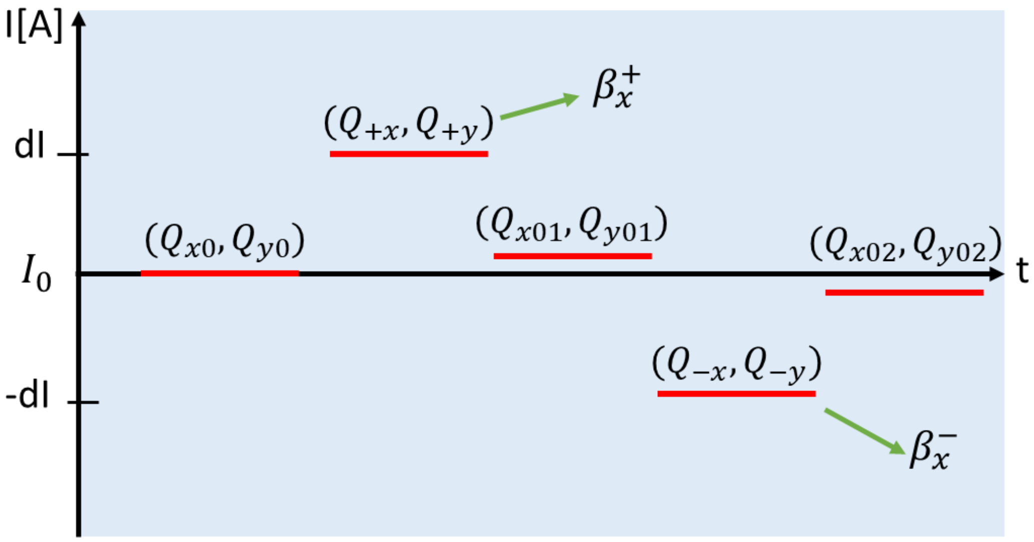


- For 60 BPMs and 5 turns.

Experimental procedure



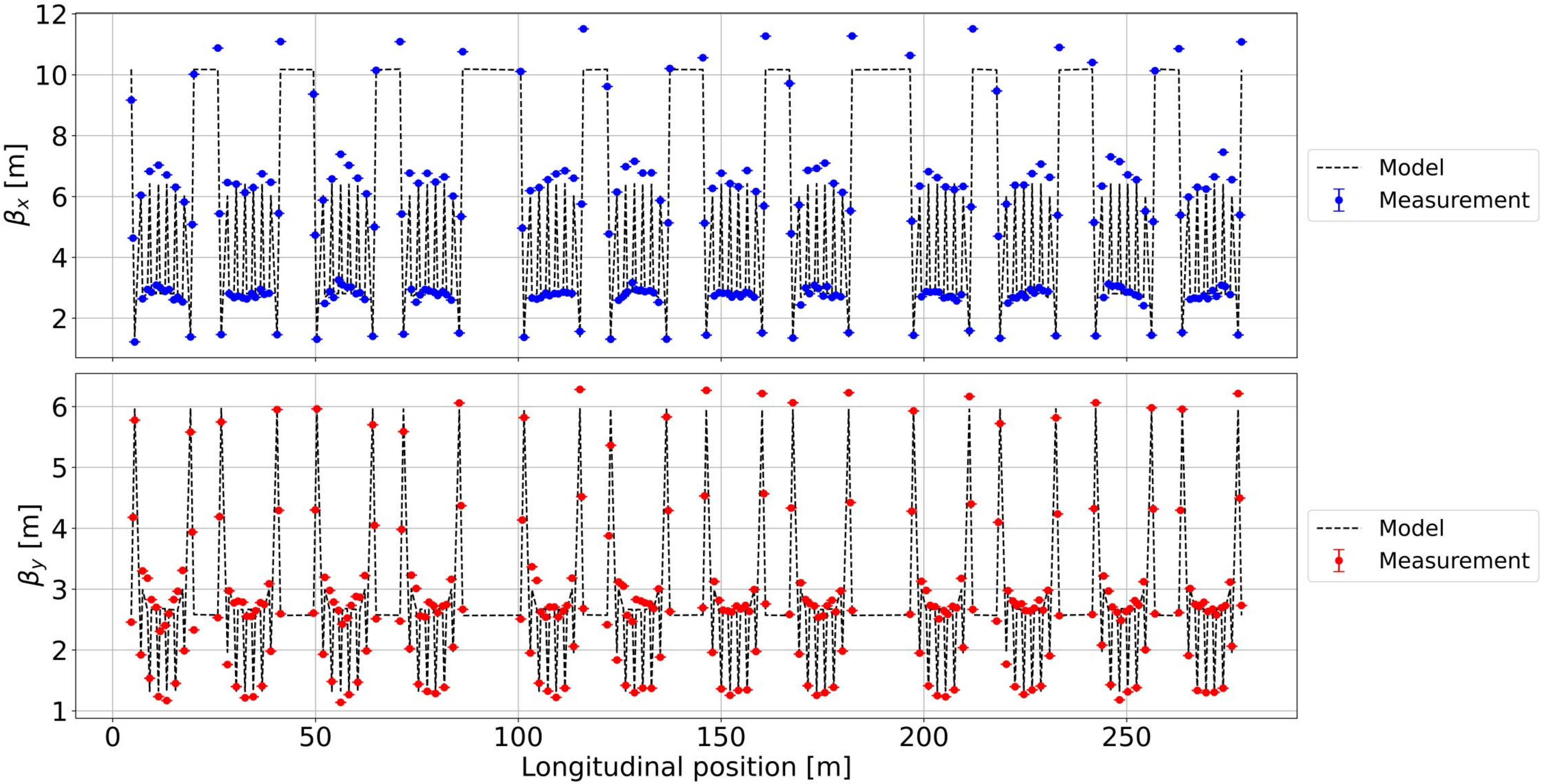
Varying the current of one quadrupole



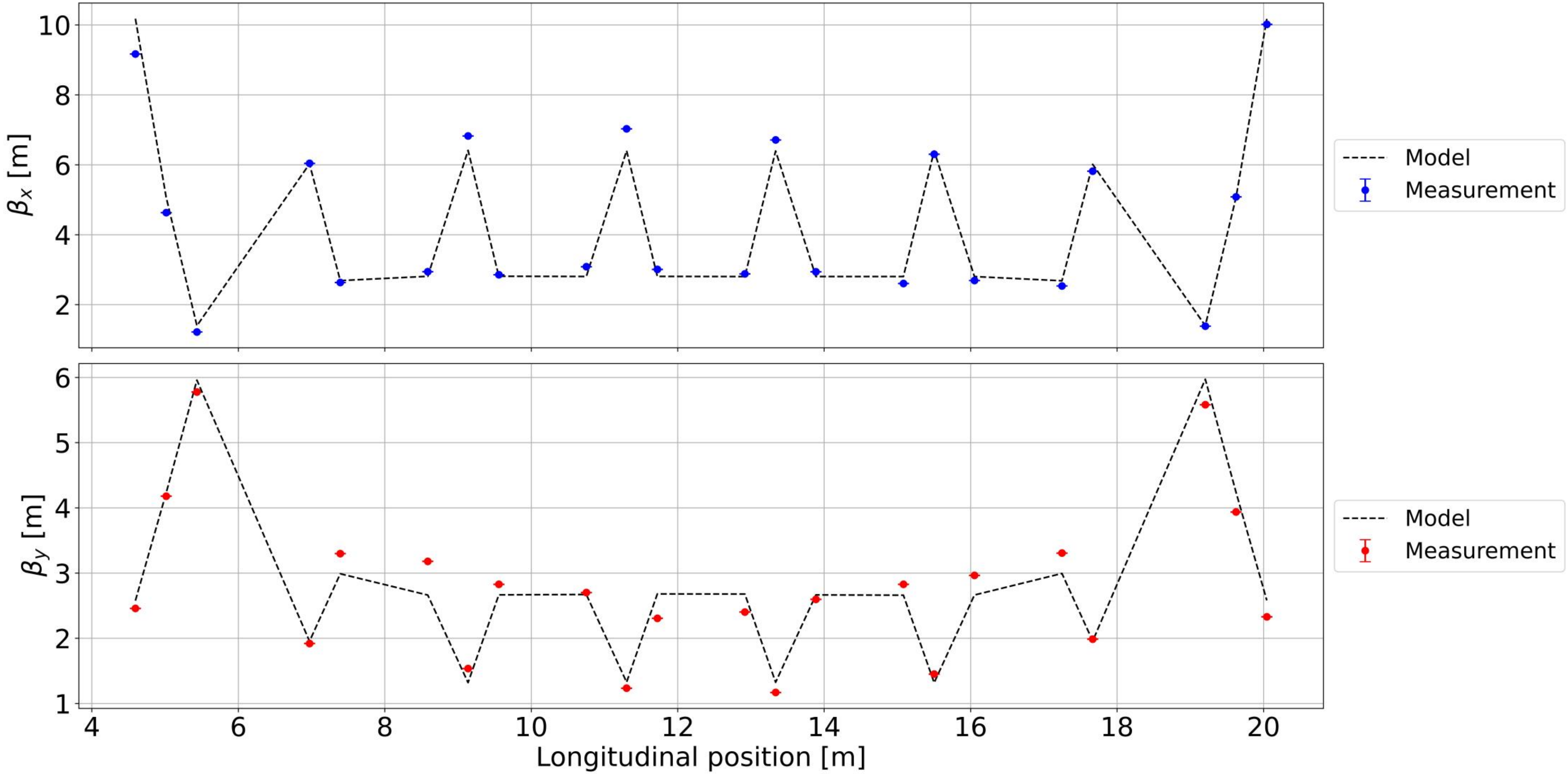
Not back to the original tune due to hysteresis

- $dI = 1\ A$ (or $\Delta K \approx 0.0072\ m^{-1}$)
- Tune measured five times and averaged
- Vary the current until $|Q_{x0}^* - Q_{x0}| \leq 1 \times 10^{-5}$
- Repeat in the 264 quadrupoles

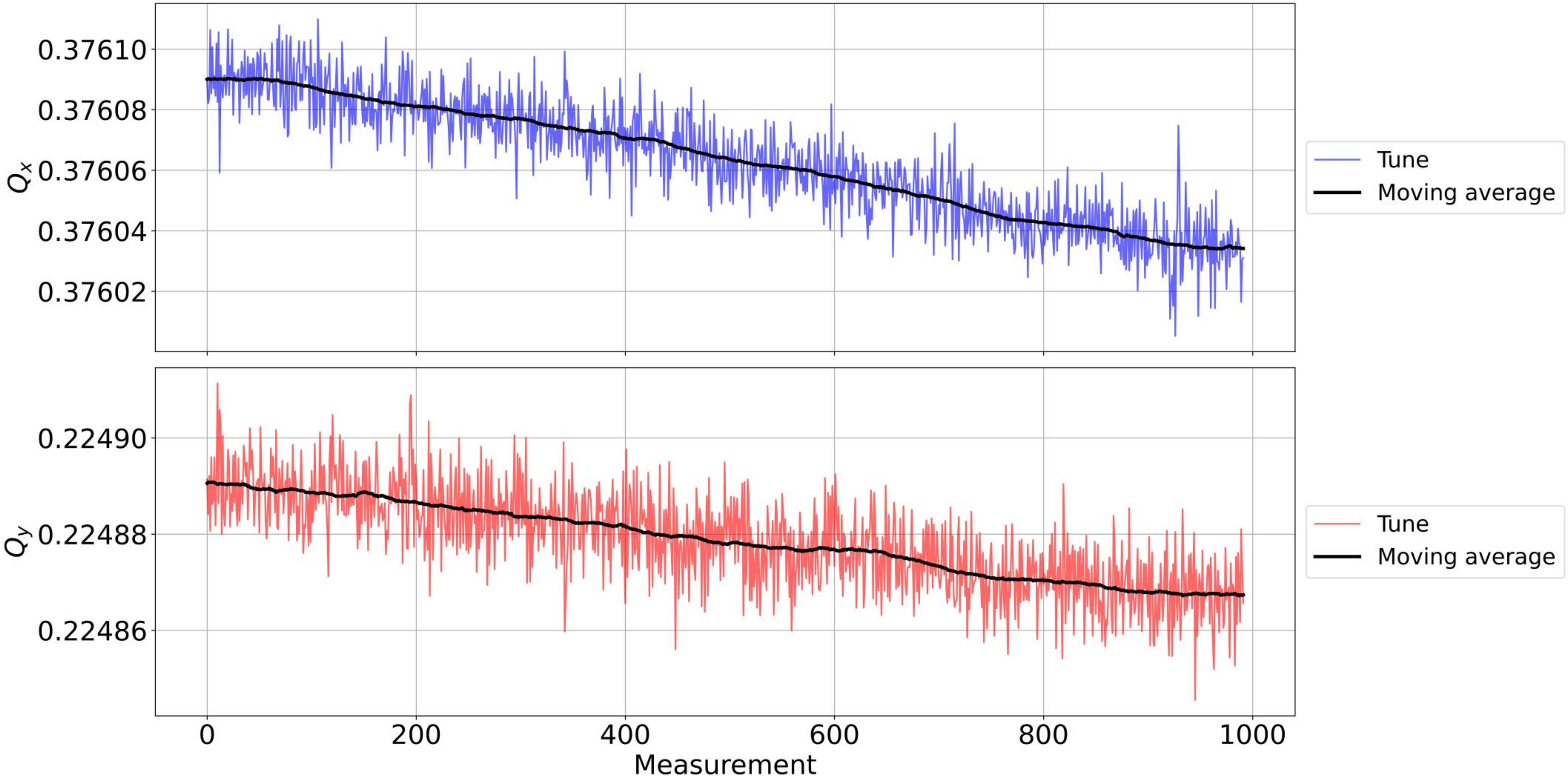
Beta function measurement



Beta function measurement

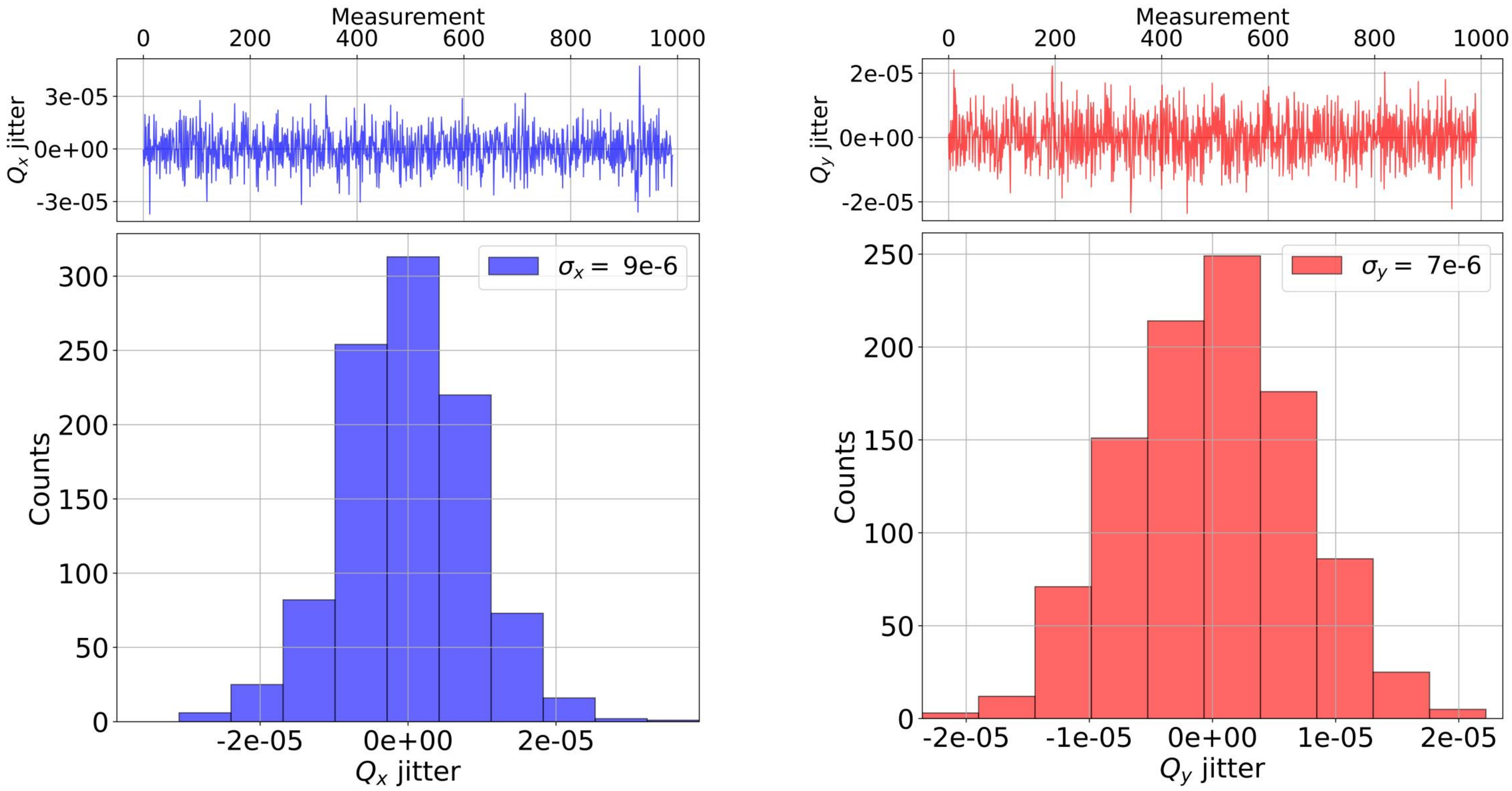


Tune noise

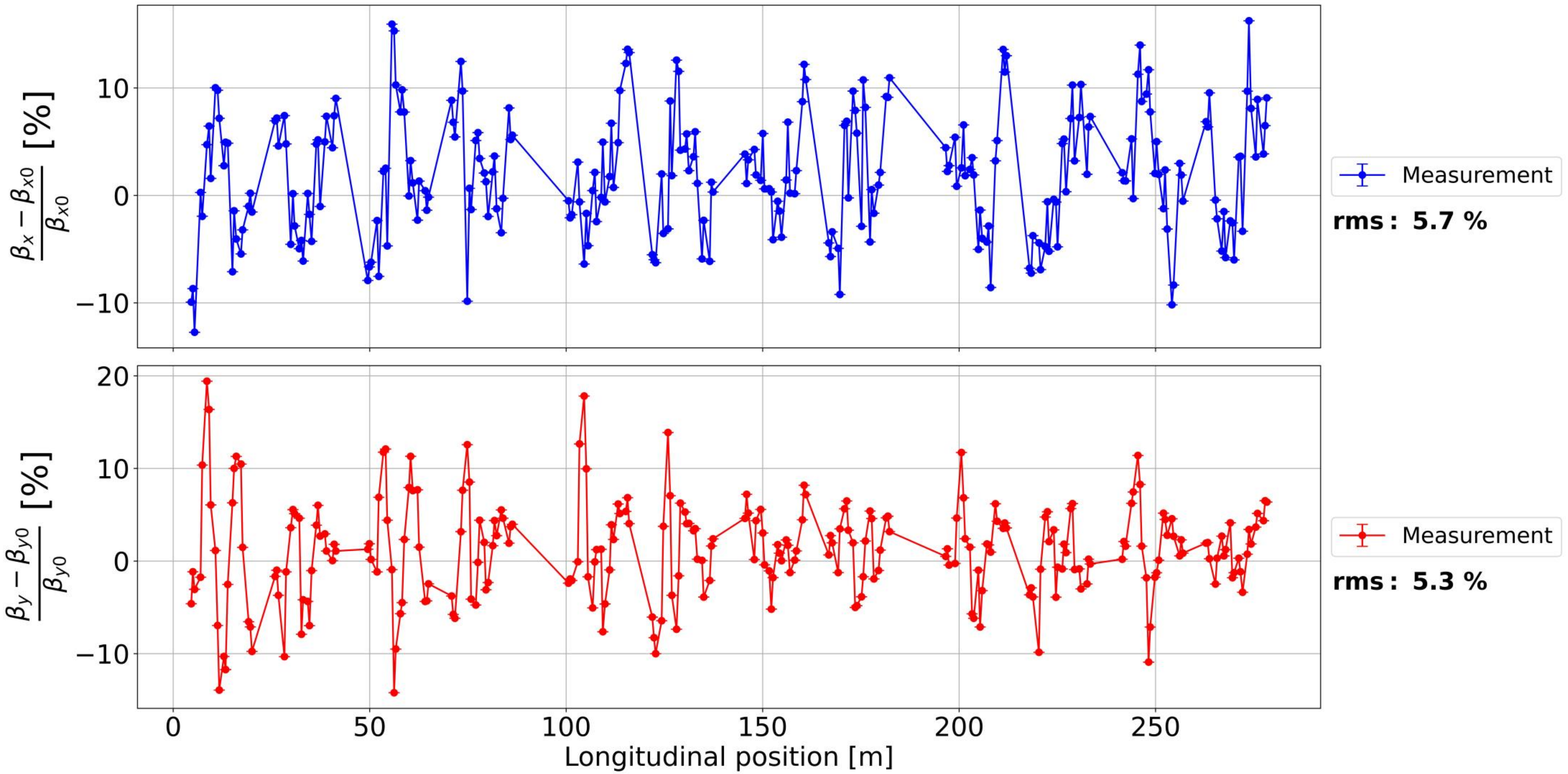


0.37610
0.37608
0.37606
0.37604
0.37602
Q_x
Tune
Moving average
0.22490
0.22488
0.22486
Q_y
Tune
Moving average
0
200
400
600
800
1000
Measurement

Tune noise

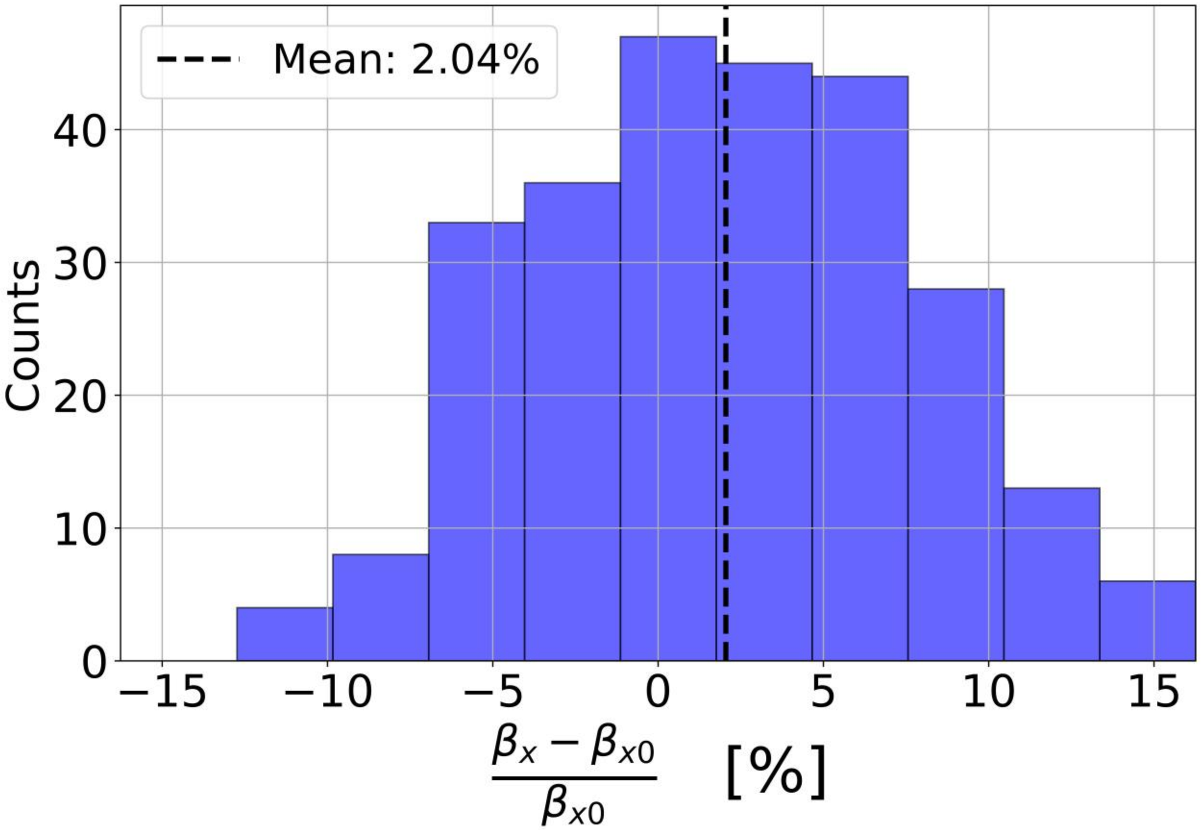

Measurement
0
200
400
600
800
1000
Q_x jitter
3e-05
0e+00
-3e-05
$\sigma_x =$ 9e-6
Counts
300
250
200
150
100
50
0
-2e-05
0e+00
2e-05
Q_x jitter
Measurement
0
200
400
600
800
1000
Q_y jitter
2e-05
0e+00
-2e-05
$\sigma_y =$ 7e-6
Counts
250
200
150
100
50
0
-2e-05
-1e-05
0e+00
1e-05
2e-05
Q_y jitter

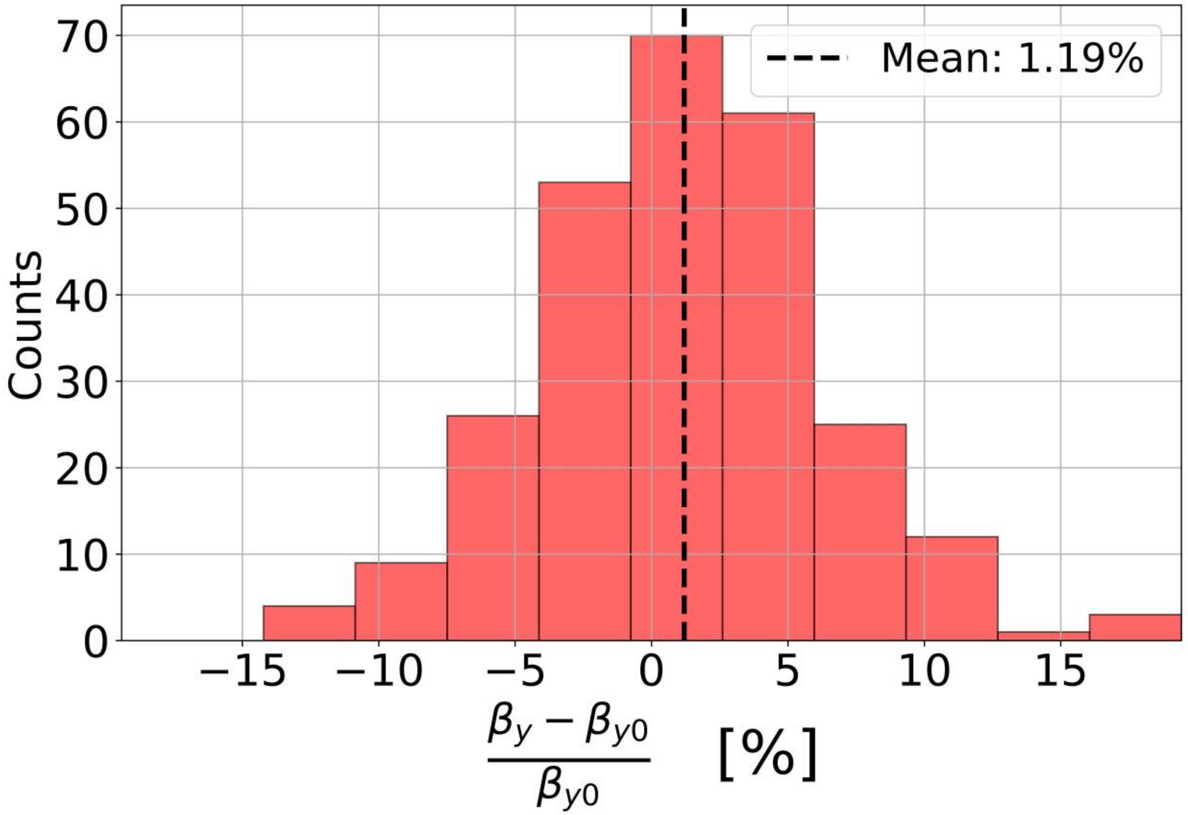
Beta-beat



Targeted rms beta-beat: <2%

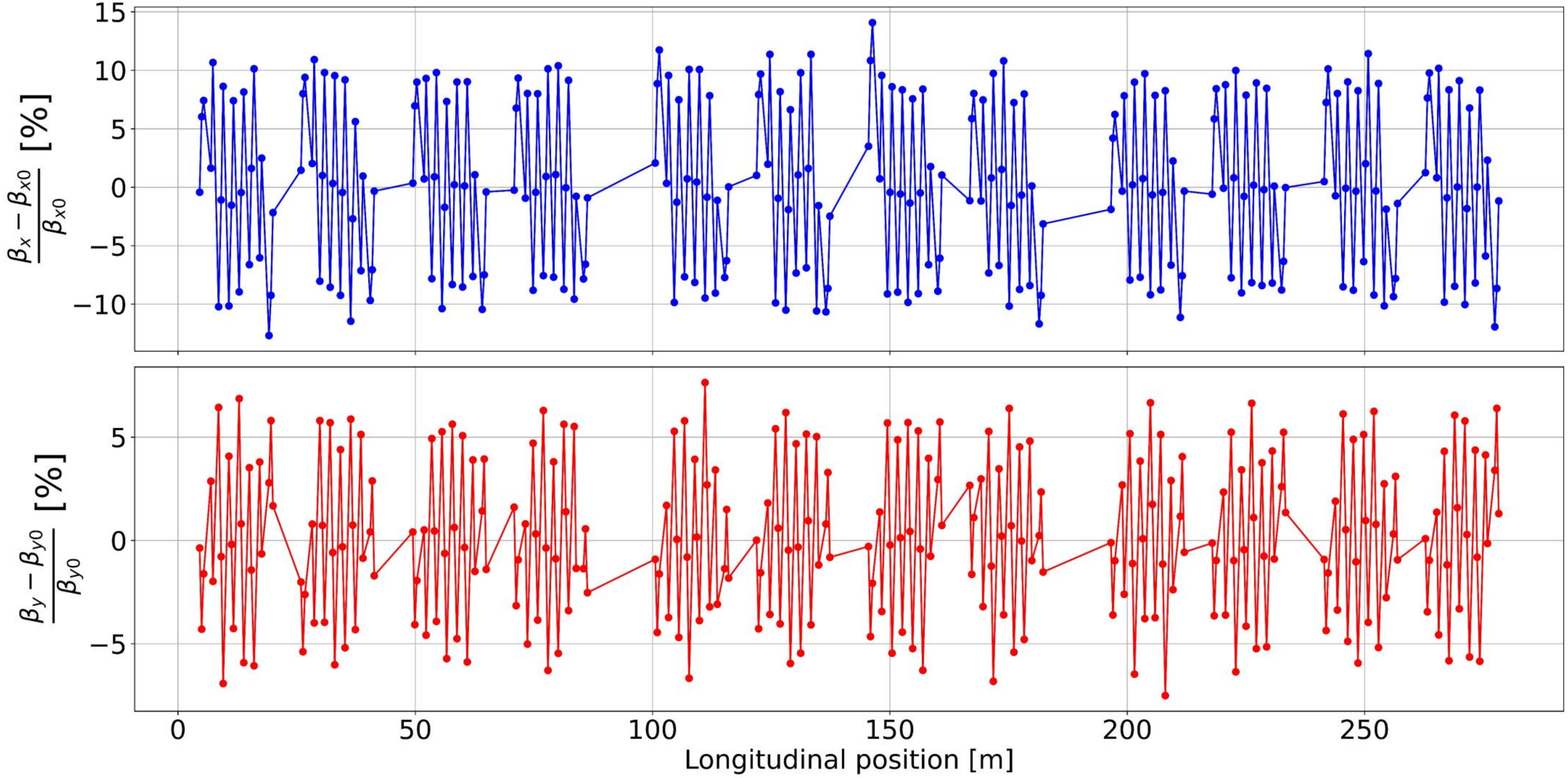
Beta-beat





- Systematic errors:
  - Magnet transfer function.
  - Closed orbit distortions.

Simulation of beta-beat



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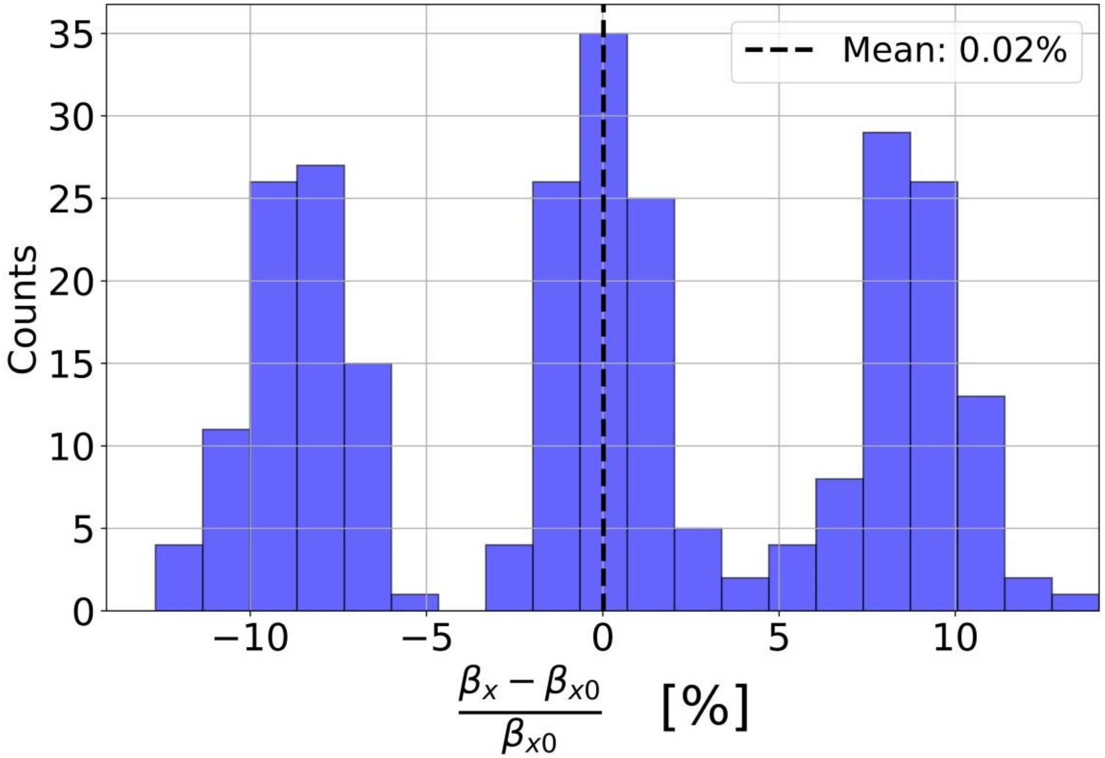
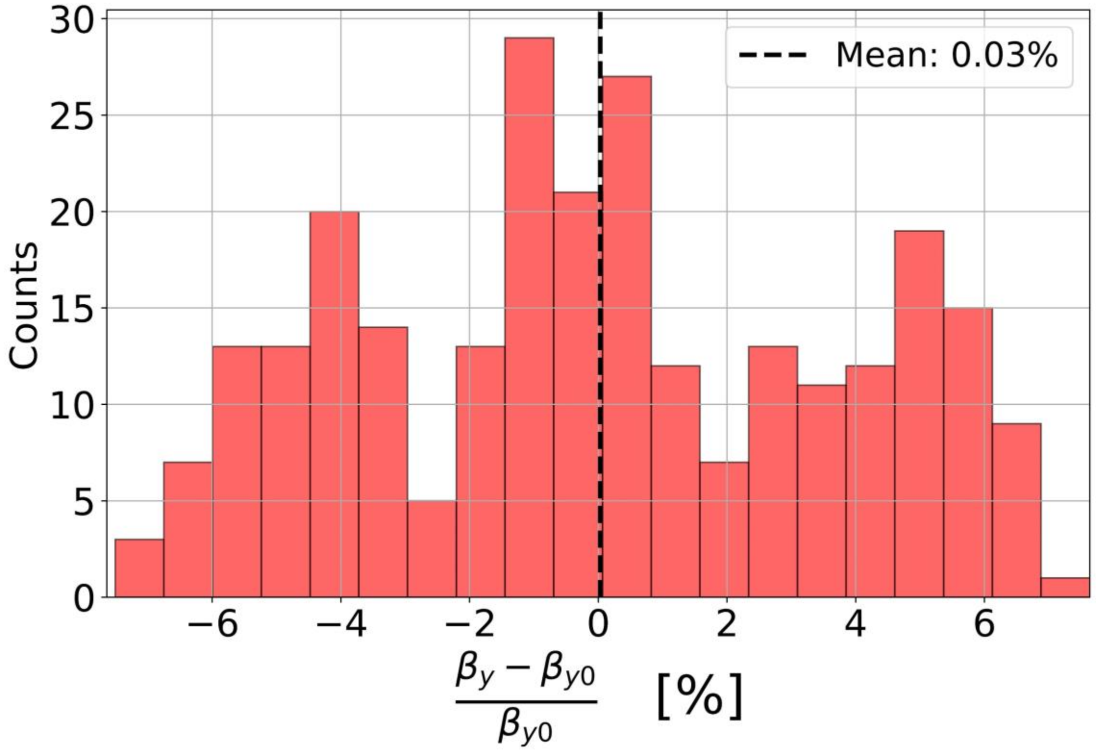
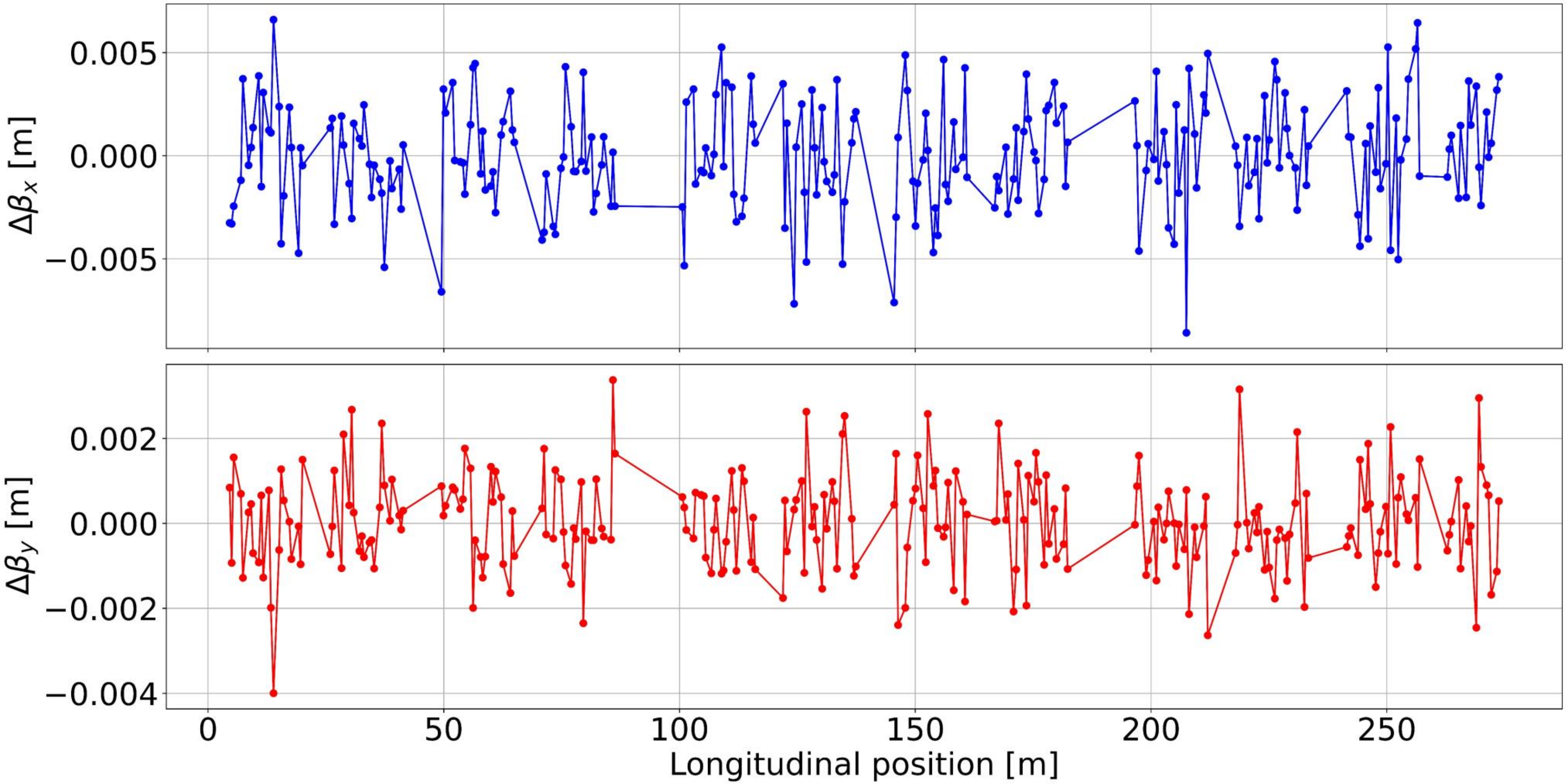
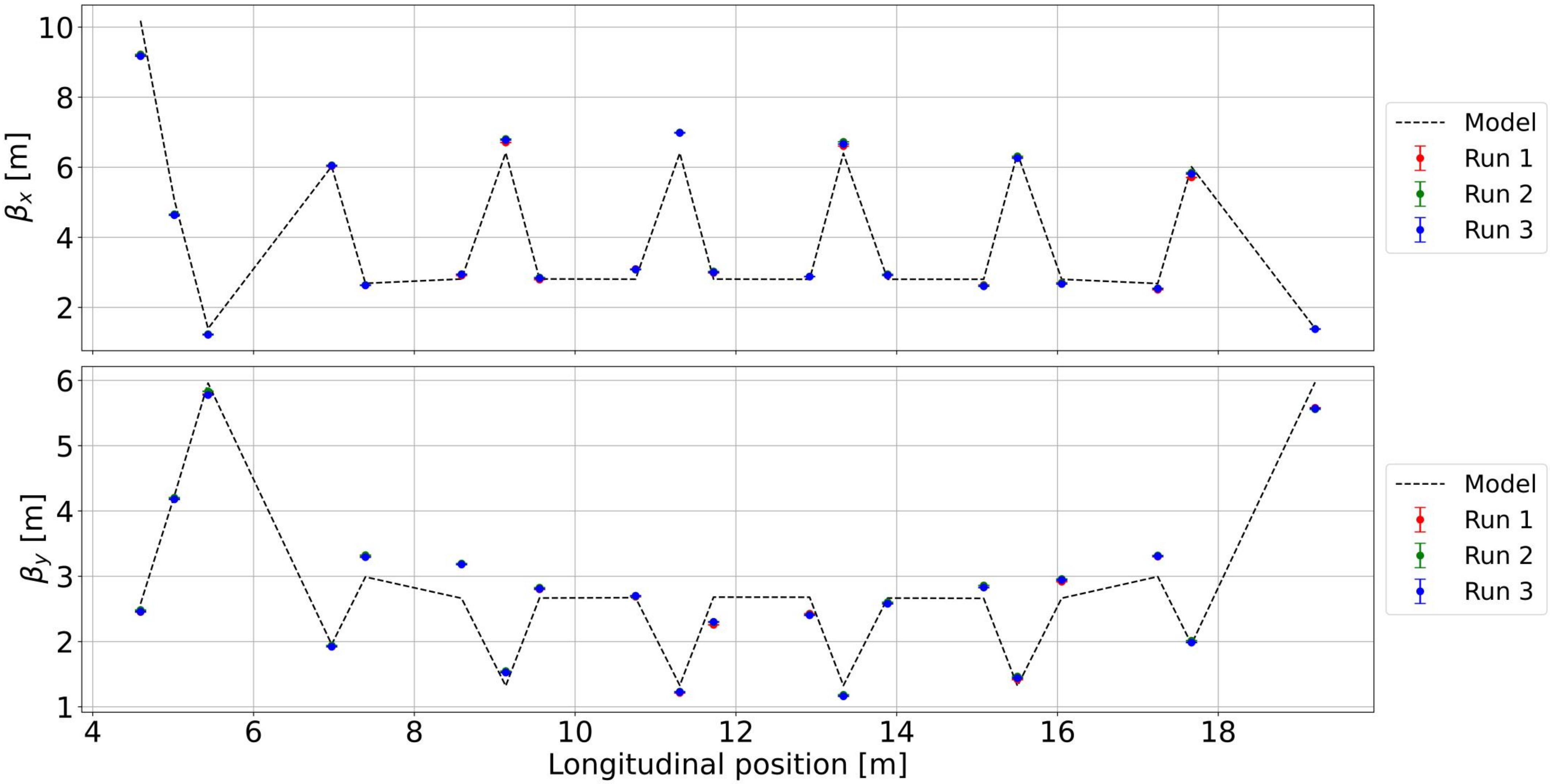




Changes in beta function due to closed orbit distortions



Reproducibility



Summary and outlook

Errors			
Type	**Origin**	**Contribution**	
		$\Delta\beta_x$ [cm]	**$\Delta\beta_y$ [cm]**
Statistical	Tune jitter	~0.55	~0.38
Systematic	Closed orbit distortions	<0.90	<0.40
	Magnet transfer function	-	-

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Optics correction

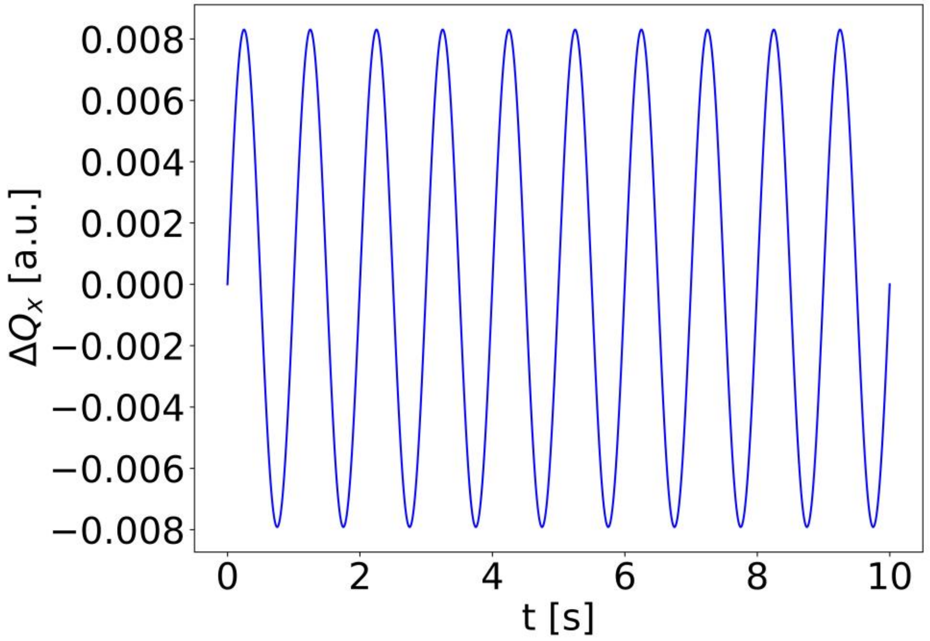
- We cannot correct the vertical and horizontal plane independently
- Errors contributing to the beta-beat are not necessarily in the quadrupoles
- We want to correct the beta-beat without perturbing the dispersion

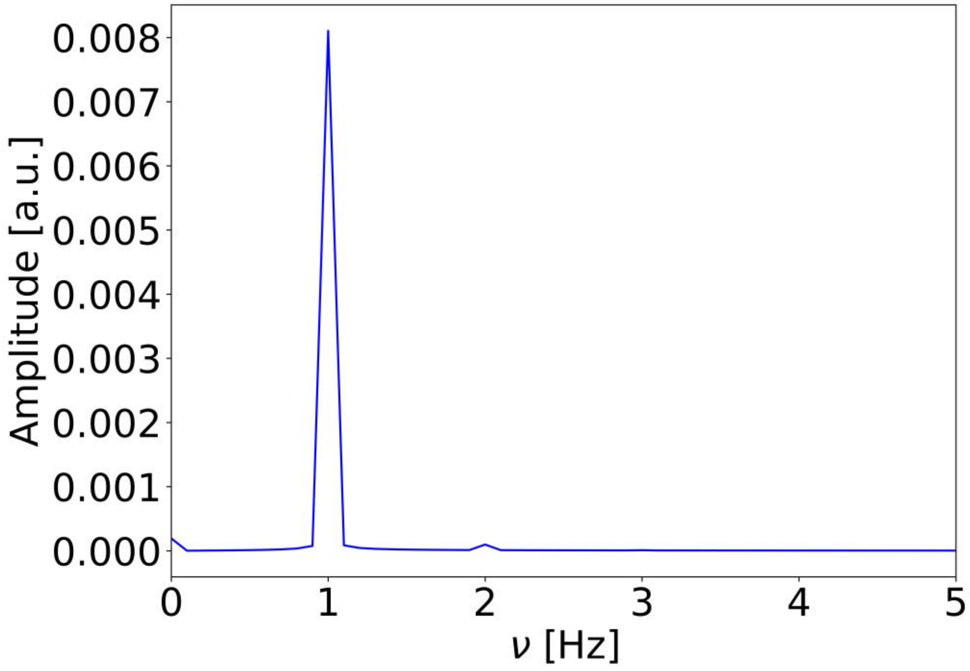
❖ 112 additional quadrupoles to help with the correction

Outlook: Sinusoidal QV

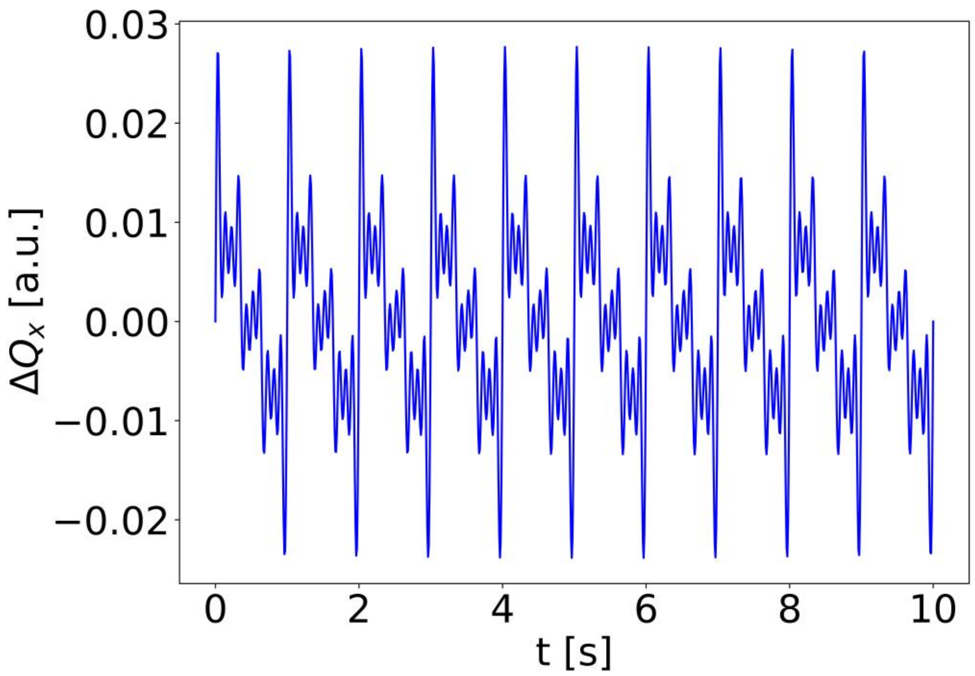
- QV is lengthy, ~2 hours for the whole ring. Working on a faster version of QV: **Sinusoidal QV**

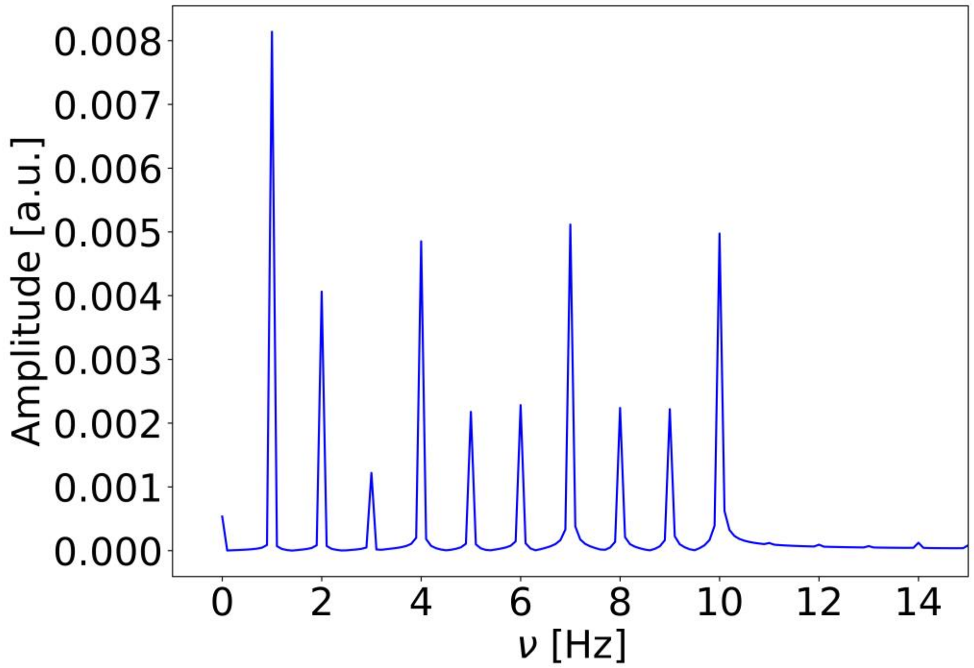
Quadrupole strength: $K = A \sin(2\pi\nu t + \phi)$





10 consecutive quadrupoles, $A = 0.01$, ν from 1 to 10 Hz, $\phi = 0$, $\mathrm{K} = A\sin(2\pi\nu t + \phi)$





References

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[1] A. Streun et al. “Swiss Light Source upgrade lattice design.” In: Phys. Rev. Accel. Beams 26 (9 Sept. 2023), p. 091601. doi: 10.1103/PhysRevAccelBeams.26.091601.

[2] G. M. Michiko and F. Zimmermann. Measurement and Control of Charged Particle Beams. Heidelberg: Springer, 2003. DOI: 10.1007/978-3-662-08581-3.

[3] P. Zisopoulos, Y. Papaphilippou, and J. Laskar. “Refined betatron tune measurements by mixing beam position data”. In: Phys. Rev. Accel. Beams 22 (7 July 2019), p. 071002. doi:10.1103 /PhysRevAccelBeams.22.071002.

[4] A. Wolski. Beam Dynamics in High Energy Particle Accelerators. London: Imperial College Press, 2014.


SINQ

Extra slides

NAFF

Consider the signal

$$w = x_{\text{norm}} - ip_{x,\text{norm}} = \sqrt{2J_x}e^{i\phi_x}$$

On the n-th turn,

$$\omega_n = \sqrt{2J_x}e^{i\phi_{x0}}e^{2\pi i n Q_x}$$

The Fourier transform is,

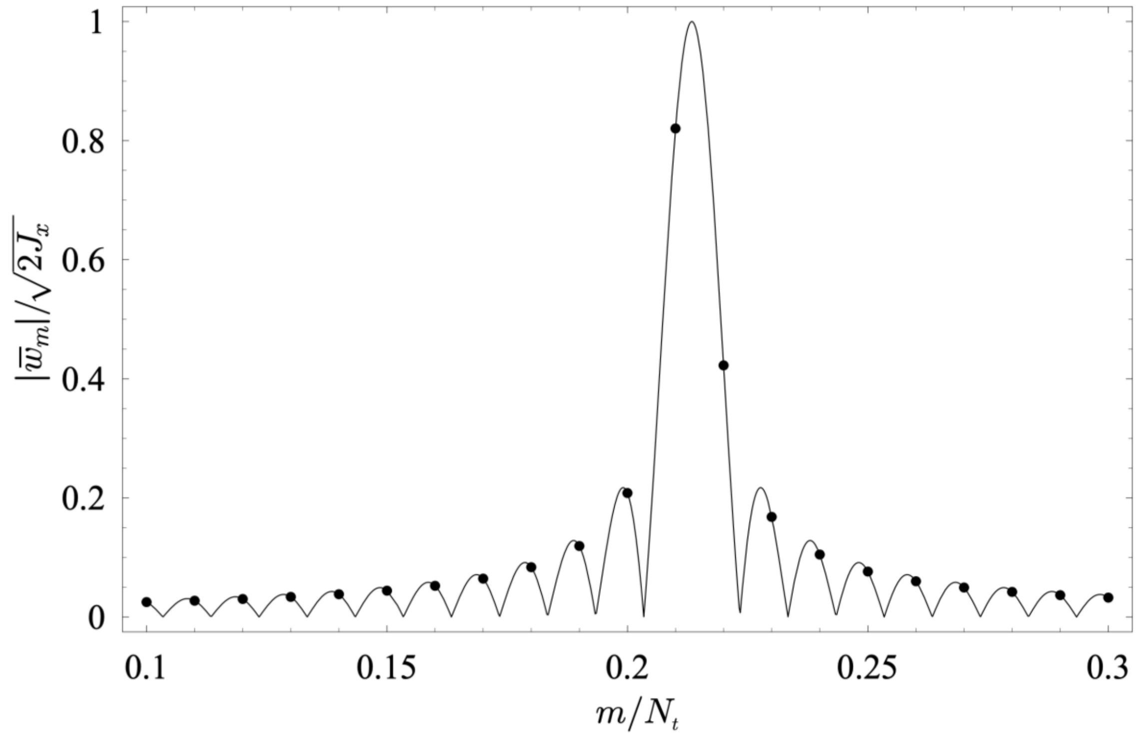
$$\bar{w}_m = \frac{1}{N}\sum_{n=0}^{N-1} e^{-2\pi i m n/N} w_n$$

$$\bar{w}_m = \sqrt{2J_x}e^{i\phi_{x0}}\frac{1-e^{2\pi i\Delta}}{1-e^{2\pi i\Delta/N}}, \quad \Delta = N\, frac(Q_x) - m$$

Let m be a real number,

$$frac(Q_x) = \frac{\hat{m}}{N}$$

[4]



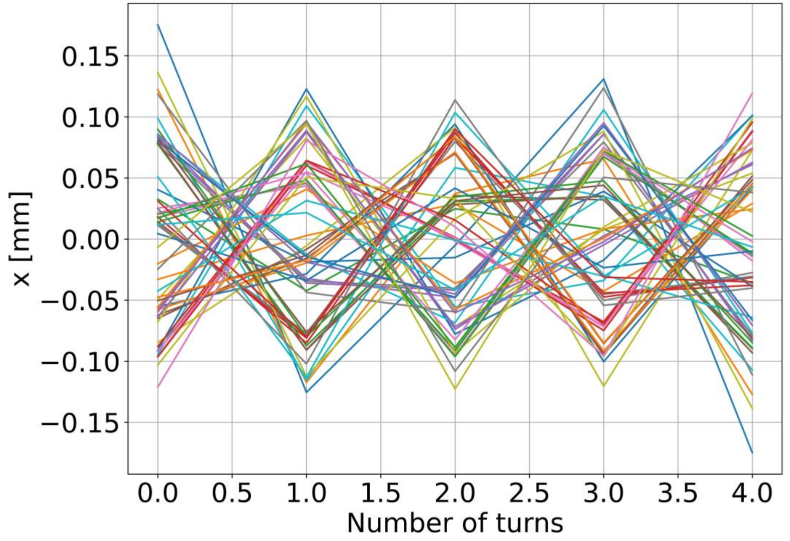
Finding an m that maximizes $|\bar{\omega}_m|$ implies searching for a frequency that gives the maximum overlap between the measured signal and a signal at the given frequency.

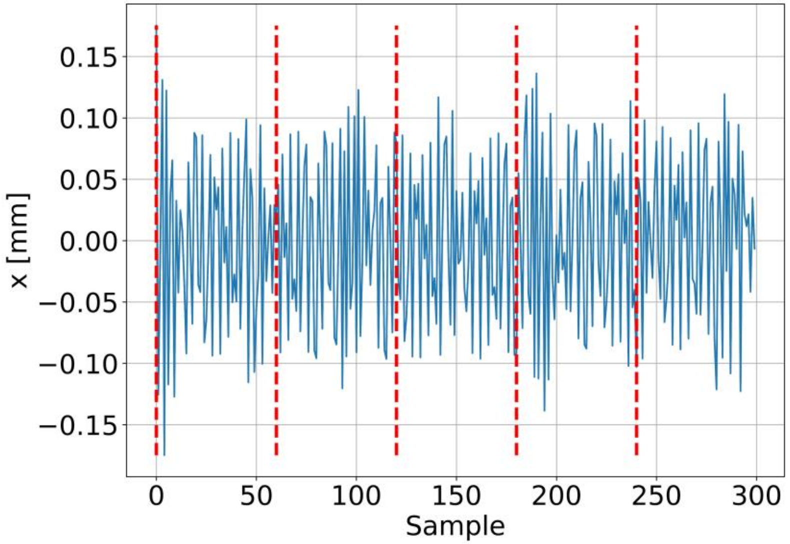
Mixed BPM Method

- Use data of M BPMs for N turns with NAFF method
- Vectorize $N \times M$ array. More samples (NM), and higher sample rate (M per turn)
- Transform the $N \times M$ array into a vector of $1 \times MN$ dimensions (BPM by BPM):

$$A = \begin{pmatrix} z_{11} & \cdots & z_{1M} \\ \cdots & \cdots & \cdots \\ z_{N1} & \cdots & z_{NM} \end{pmatrix} \longrightarrow \tilde{A} = (z_{11} z_{12} \cdots z_{NM-1} z_{NM})$$

- Error for N turns: $\varepsilon(N) = |Q(N) - Q_0| \propto \frac{1}{N^l}$
- FFT: $l = 1$
- NAFF (Hann window order p): $l = 2p + 2$
- Mixed BPM method: $\varepsilon(N) \propto \frac{1}{M^{2p+1} N^{2p+2}}$

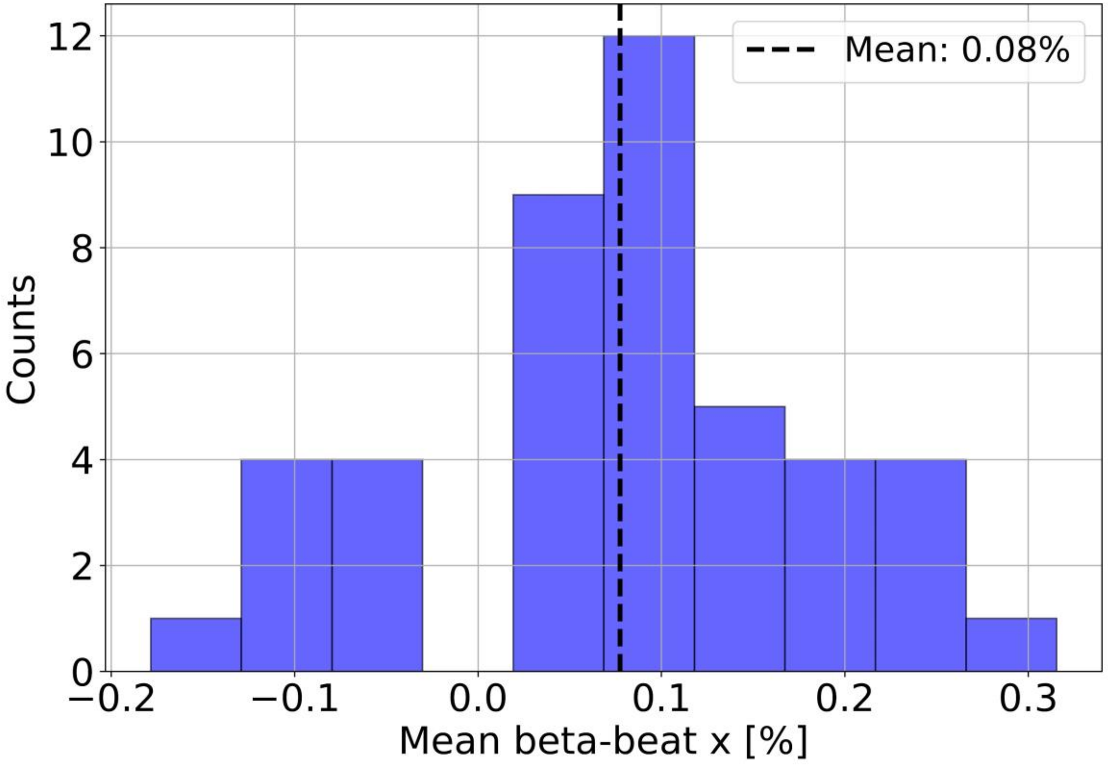


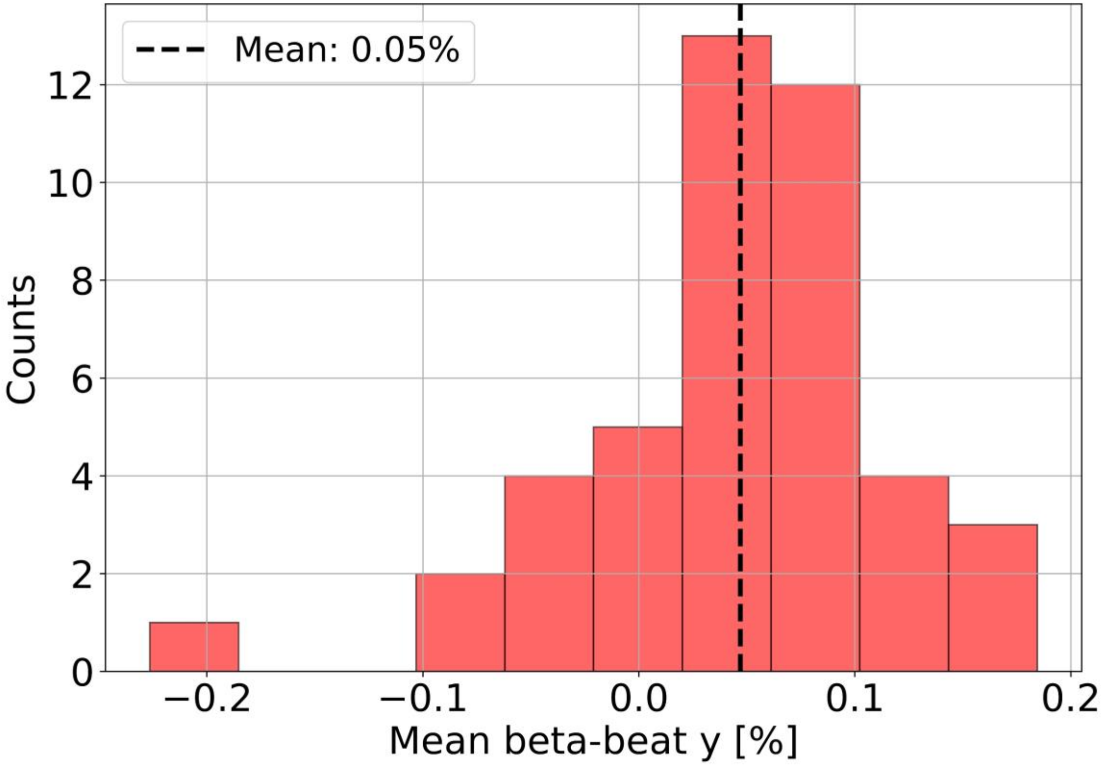


[3]

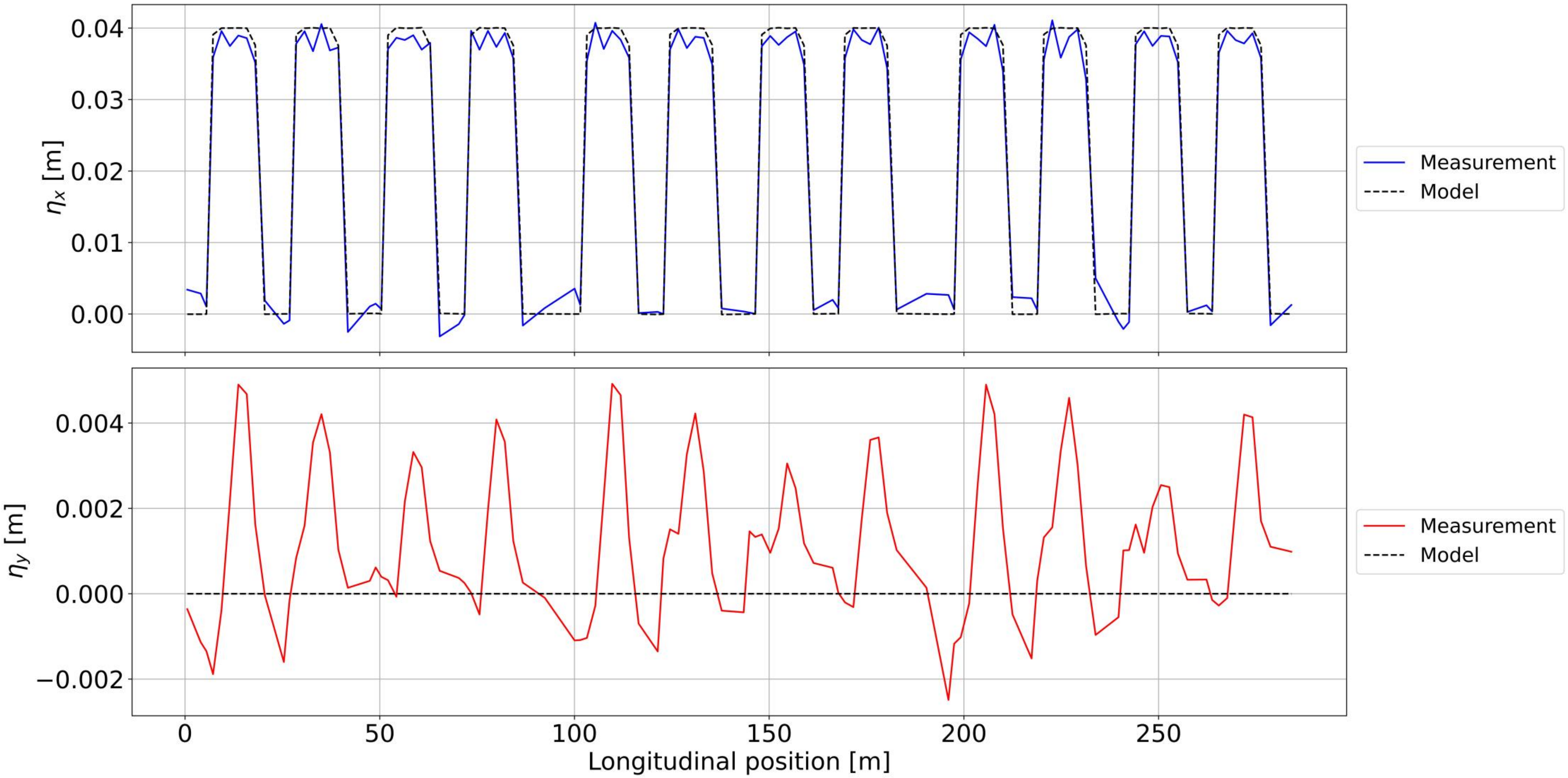
Simulation of beta-beat

- Distribution of the mean beta-beat for 44 seeds.





Dispersion



0.04
0.03
0.02
0.01
0.00
η_x [m]
Measurement
Model
0.004
0.002
0.000
−0.002
η_y [m]
Measurement
Model
0
50
100
150
200
250
Longitudinal position [m]

Optics correction

Consider the *n* optics function $f_1, f_2, \dots, f_i, \dots, f_n$ and *n* quadrupoles with strengths $k_1, k_2, \dots, k_i, \dots, k_n$.

$$f_i = f_i(k_1, k_2, \dots, k_i, \dots, k_n)$$

We want to reach the ideal values $f_{1,s}, f_{2,s}, \dots, f_{n,s}$. We have the initial values $f_{1,0}, f_{2,0}, \dots, f_{n,0}$ and $k_{1,0}, k_{2,0}, \dots, k_{n,0}$. We can do a first order expansion,

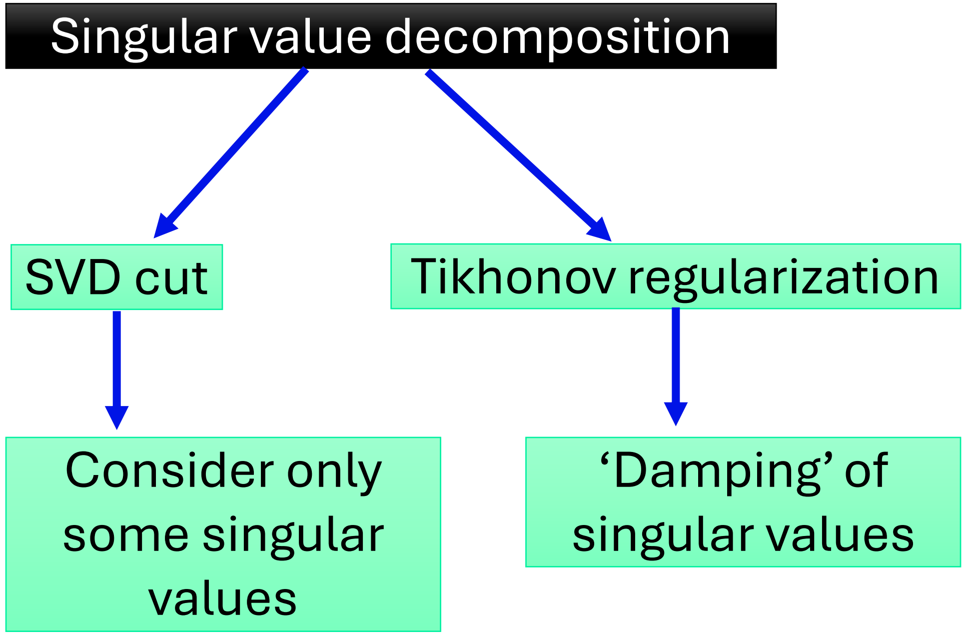
$$\begin{pmatrix} f_{1,s} \\ f_{2,s} \\ \vdots \\ f_{n,s} \end{pmatrix} - \begin{pmatrix} f_{1,0} \\ f_{2,0} \\ \vdots \\ f_{n,0} \end{pmatrix} = \begin{pmatrix} \frac{\partial f_1}{\partial k_1} & \frac{\partial f_1}{\partial k_2} & \cdots & \frac{\partial f_1}{\partial k_n} \\ \frac{\partial f_2}{\partial k_1} & \frac{\partial f_2}{\partial k_2} & \cdots & \frac{\partial f_2}{\partial k_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial k_1} & \frac{\partial f_n}{\partial k_2} & \cdots & \frac{\partial f_n}{\partial k_n} \end{pmatrix} \begin{pmatrix} k_1 - k_{1,0} \\ k_2 - k_{2,0} \\ \vdots \\ k_n - k_{n,0} \end{pmatrix} = A(\vec{k} - \vec{k}_0)$$

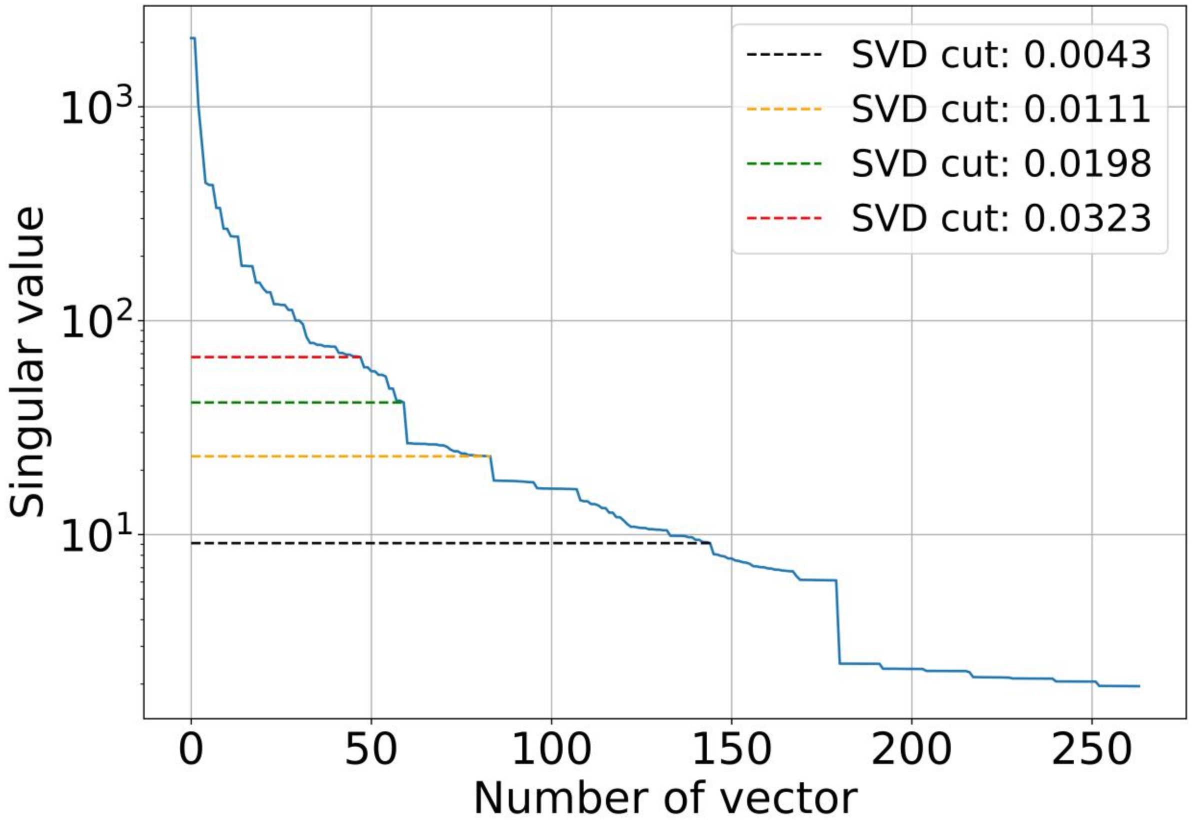
A is the response matrix.

Unfortunately, we would like match m optics function with only n quadrupoles, $m > n$.

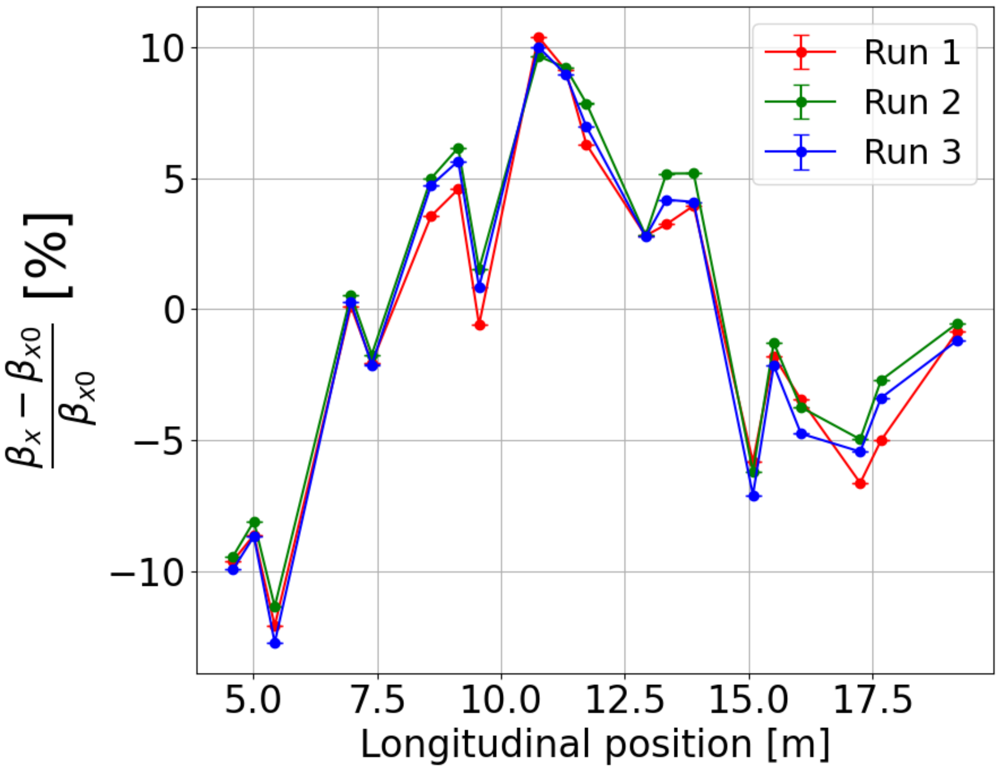
→ Singular value decomposition

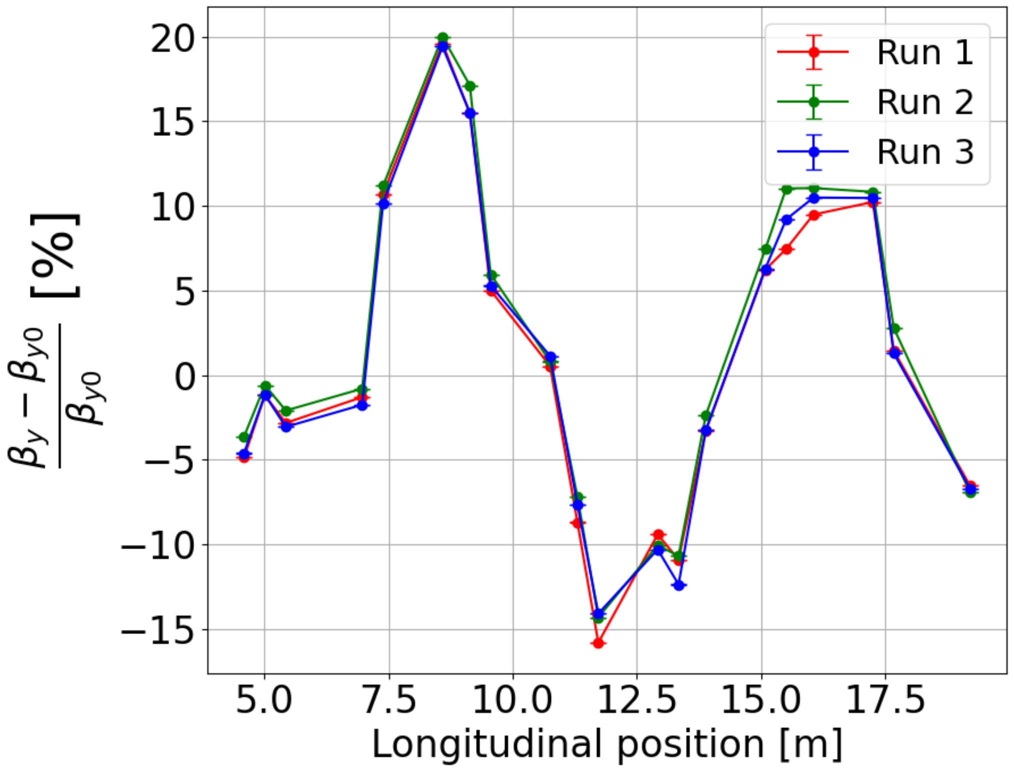
Optics correction

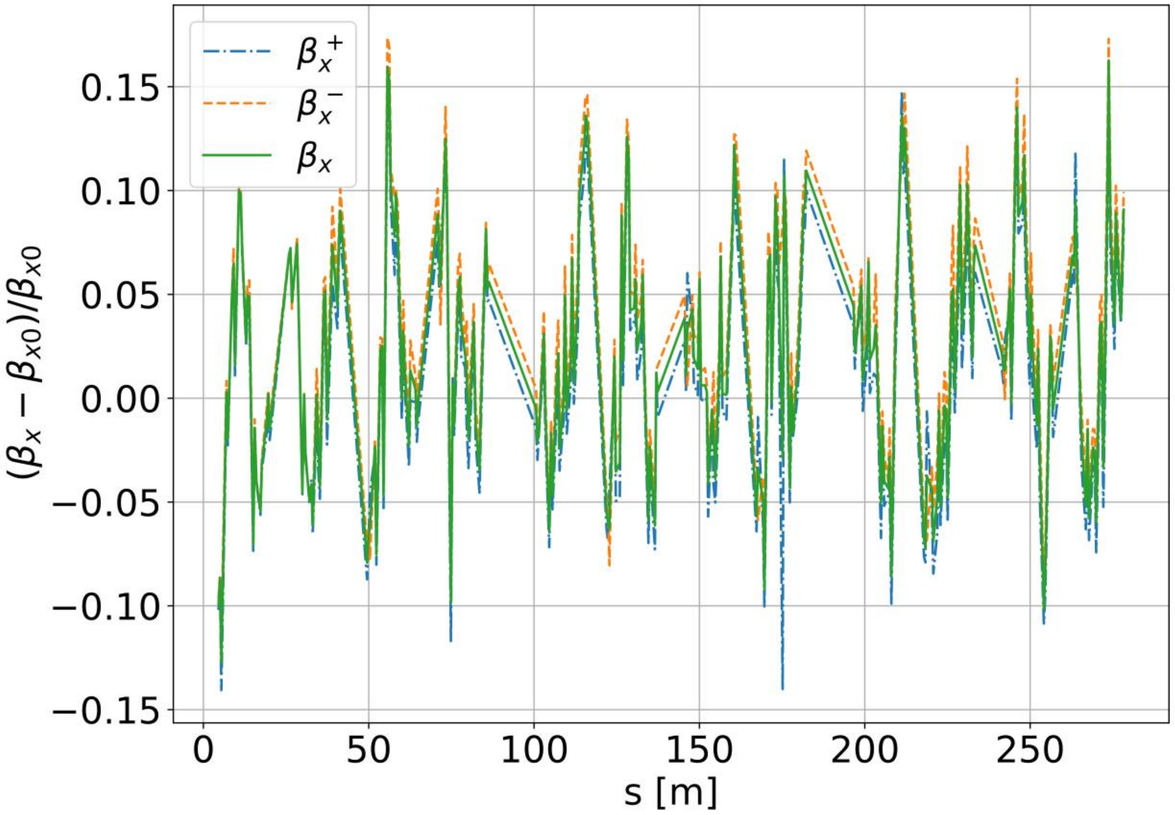


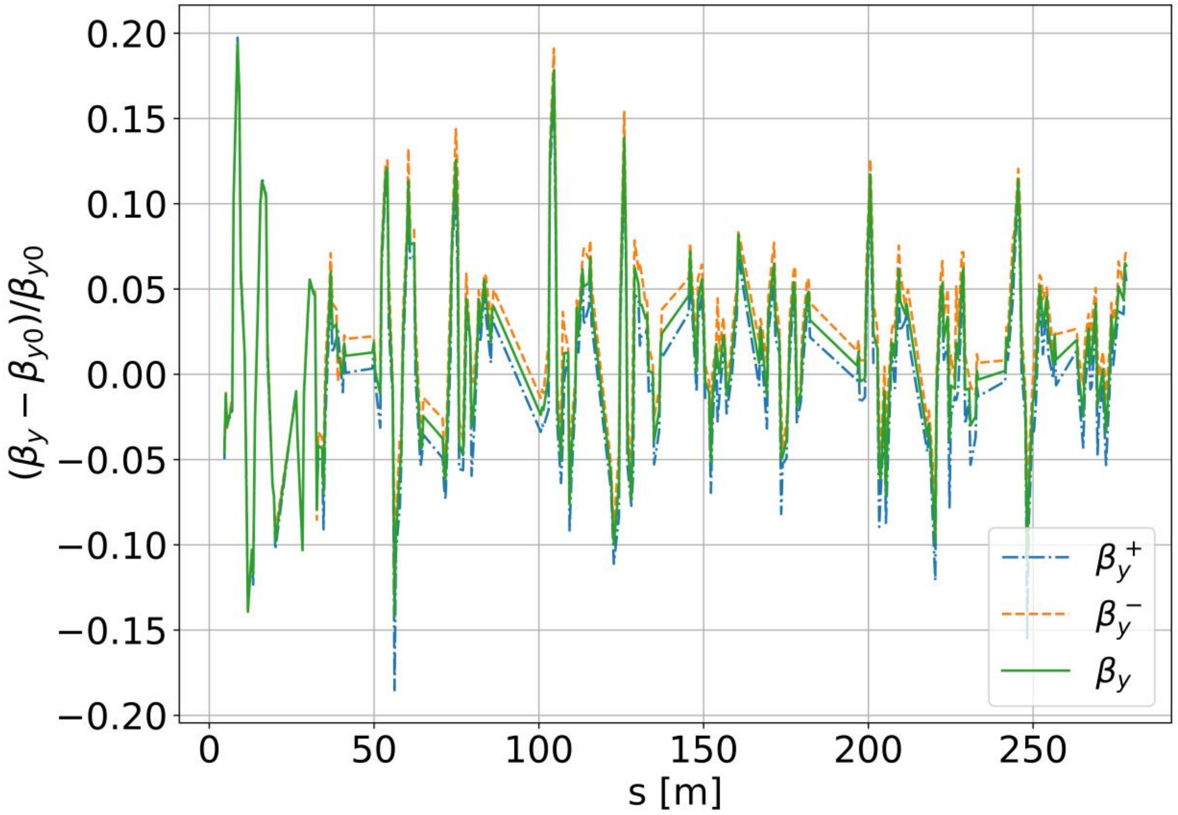


Reproducibility






β_x^+
β_x^-
β_x
$(\beta_x - \beta_{x0})/\beta_{x0}$
s [m]


β_y^+
β_y^-
β_y
$(\beta_y - \beta_{y0})/\beta_{y0}$
s [m]

Average beta function

$$\beta_{\text{meas; } x,y} = \frac{1}{L_q}\int_{L_q} \beta_{x,y} ds$$

$$\begin{pmatrix} x_1 \\ x_1' \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_0 \\ x_0' \end{pmatrix}$$

$$A_{\text{quad}} = \begin{pmatrix} \cos\left(L_q\sqrt{|k|}\right) & \frac{\sin\left(L_q\sqrt{|k|}\right)}{\sqrt{|k|}} \\ -\sqrt{|k|}\sin\left(L_q\sqrt{|k|}\right) & \cos\left(L_q\sqrt{|k|}\right) \end{pmatrix}$$

The Twiss parameters can also be transported,

$$\begin{pmatrix} \beta_1 \\ \alpha_1 \\ \gamma_1 \end{pmatrix} = \begin{pmatrix} a_{11}^2 & -2a_{11}a_{12} & a_{12}^2 \\ -a_{11}a_{21} & 2a_{12}a_{21} & -a_{12}a_{22} \\ a_{21}^2 & -2a_{21}a_{22} & a_{22}^2 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0, \end{pmatrix}$$

$$\beta_1 = a_{11}^2\beta_0 - 2a_{11}a_{12}\alpha_0 + a_{12}^2\gamma_0$$

Average beta function

$$\beta_1 = \cos^2\left(L_q\sqrt{|k|}\right)\beta_0 - 2\frac{1}{\sqrt{|k|}}\cos\left(L_q\sqrt{|k|}\right)\sin\left(L_q\sqrt{|k|}\right)\alpha_0 + \frac{1}{|k|}\sin^2\left(L_q\sqrt{|k|}\right)\gamma_0$$

$$= \cos^2\left(L_q\sqrt{|k|}\right)\beta_0 - \frac{1}{\sqrt{|k|}}\sin\left(2L_q\sqrt{|k|}\right)\alpha_0 + \frac{1}{|k|}\sin^2\left(L_q\sqrt{|k|}\right)\gamma_0,$$

$$L_q\sqrt{|k|} \ll 1,$$

$$\beta_1 \approx \beta_0 - \frac{2L_q\sqrt{|k|}}{\sqrt{|k|}}\alpha_0 + \frac{L_q^2|k|}{|k|}\gamma_0 = \beta_0 - 2L_q\alpha_0 + L_q^2\frac{1+\alpha^2}{\beta} \approx \beta_0 - 2L_q\alpha_0,$$

$$\beta_{\mathrm{meas};x,y} = \frac{1}{L}\int_0^L (\beta_0 - 2s\alpha_0)ds = \beta_0 - L\alpha_0 = \beta(L/2)$$

The measured beta function is equivalent to the beta function in the middle of the quadrupole. In SLS 2.0, $L_q \approx 0.0601\ m$, and the maximum magnet strength is $K = 0.037\ m^{-1}, L_q K = 0.047 \ll 1.$

Equation for beta function determination

- For the horizontal motion of a particle in a quadrupole of strength k $[m^{-2}]$:

$$\frac{d^2x}{ds^2} = -kx$$

- The effect of the quadrupole can be represented by a deflection:

$$\Delta x' = -Kx$$

With $K = k\, l_q$ the integrated quadrupole strength in m^{-1}.

- Recall the one-turn matrix:

$$R_{ii} = \begin{pmatrix} \cos(2\pi Q_{x,y}) + \alpha_{x,y}\sin(2\pi Q_{x,y}) & \beta_{x,y}\sin(2\pi Q_{x,y}) \\ -\sin(2\pi Q_{x,y}) & \cos(2\pi Q_{x,y}) - \alpha_{x,y}\sin(2\pi Q_{x,y}) \end{pmatrix} \quad (1)$$

With trace $2\cos(2\pi Q)$.

- The effect of a change in gradient can be written as:

$$\begin{pmatrix} 1 & 0 \\ -(\pm\Delta K) & 1 \end{pmatrix} \quad (2)$$

- Let $\bar{Q}_{x,y} = Q_{x,y} + \Delta Q_{x,y}$, the new tune after a change in gradient. The trace of the product of (1) and (2) should be equal to $2\cos(2\pi\bar{Q})$,

$$2\cos\left[2\pi(Q_{x,y} + \Delta Q_{x,y})\right] = 2\cos\left(2\pi Q_{x,y}\right) - \beta_{x,y}(\pm\Delta K)\sin\left(2\pi Q_{x,y}\right)$$

$$\beta_{x,y} = \pm\frac{2}{\Delta K}\left[\cot(2\pi Q_{x,y})\left\{1 - \cos(2\pi\Delta Q_{x,y})\right\} + \sin(2\pi\Delta Q_{x,y})\right]$$

If $2\pi Q_{x,y} \ll 1$ and $\cot\left(2\pi Q_{x,y}\right) \leq 1$,

$$\beta_{x,y} \approx \pm 4\pi\frac{\Delta Q_{x,y}}{\Delta K}$$

Approximation not valid close to integer and half-integer resonances and large ΔK.