Non-Redudant Aperture Interferometry for Full Transverse Beam Characterization

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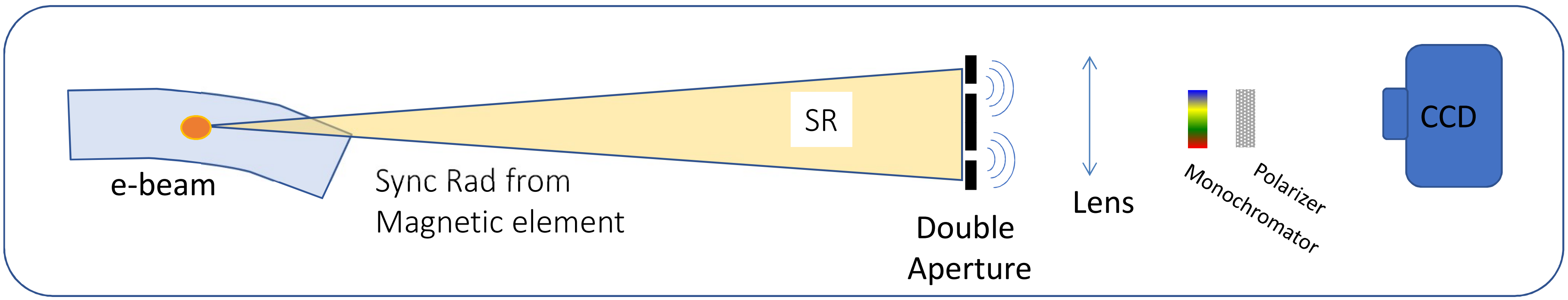
ESLS Workshop
Oct. 2025

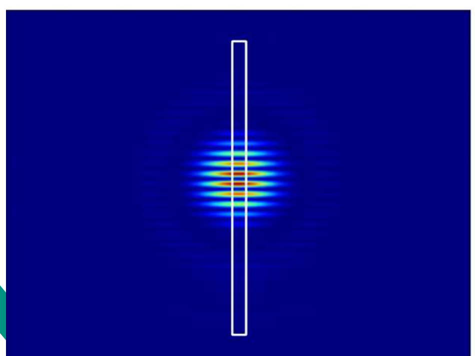
- Introduction: 2-apertures Interferometry
- Fourier Domain: from 2-holes to N-holes Interferometry
- Non-Redundant Aperture Interferometry: Self-Calibration
- Closure Amplitude Technique
- Other Applications
- Conclusions

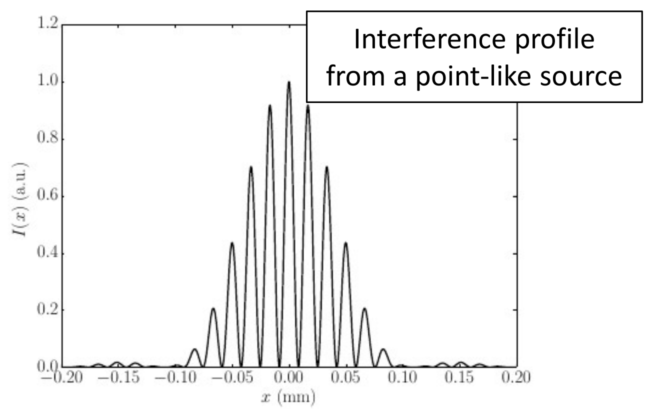
Double Aperture Interferometry

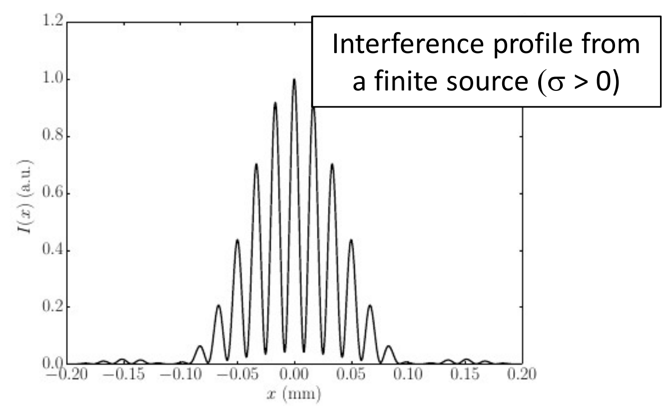
- Pioneer implementation by T. Mitsuhashi based on Young's Interferometer (~90s)
- Use the synchrotron radiation and a double-aperture mask to produce an interferogram and measure the 1st order **spatial coherence**.
- Typically used in the visible range.



Interferogram @Image Plane

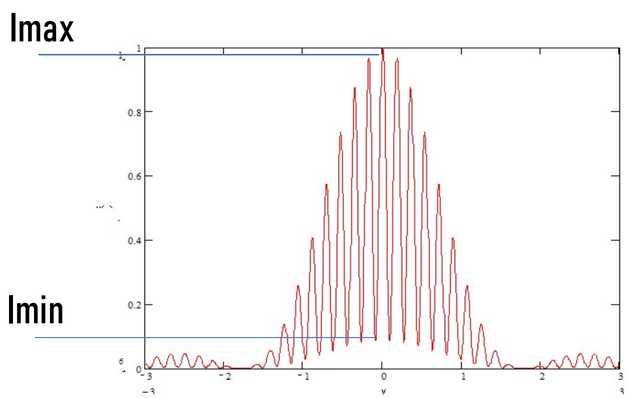






Double Aperture Interferometry

The interference pattern in the image plane follows an **analytical expression**:



$$I = I_0 \left\{ \frac{J_1\left(\frac{2\pi a x}{\lambda f}\right)}{\left(\frac{2\pi a x}{\lambda f}\right)} \right\}^2 \times \left\{1 + V \cos\left(\frac{2\pi D x}{\lambda f}\right)\right\}$$

- I_0 : intensity
- a: pinhole radius
- D: distance between pinholes
- λ = wavelength
- f: focal length
- L: distance to the source

- All parameters are known, except the "Visibility" (V)
- V obtained by fitting, or simply measuring max intensity and adjacent minima.

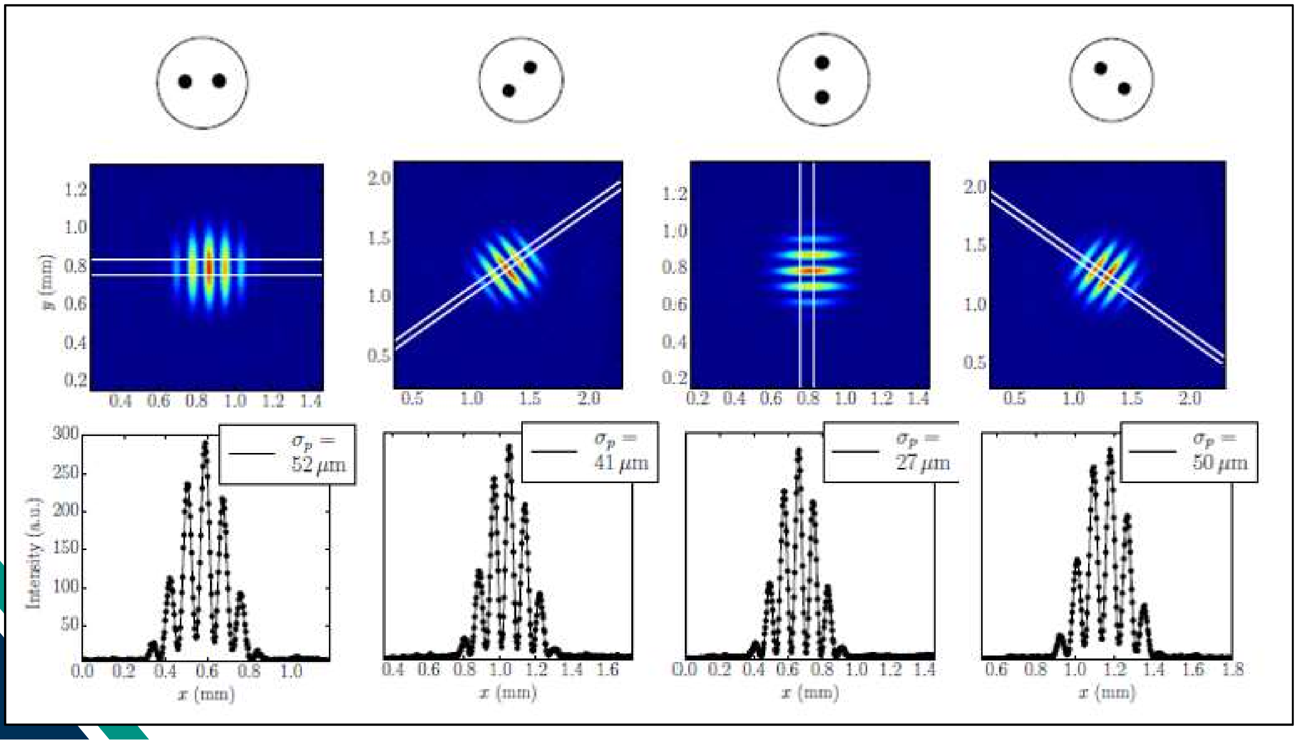
$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

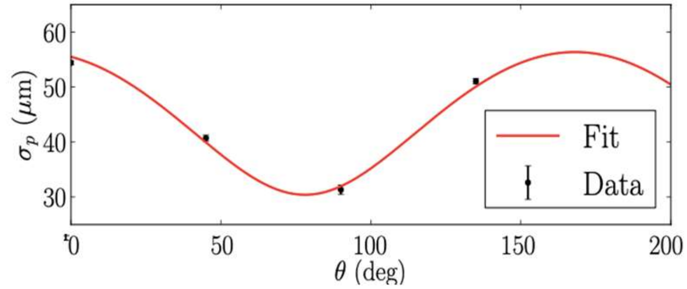
- From V, the beam size (σ_x) is obtained using the Van Cittern-Zernique theorem

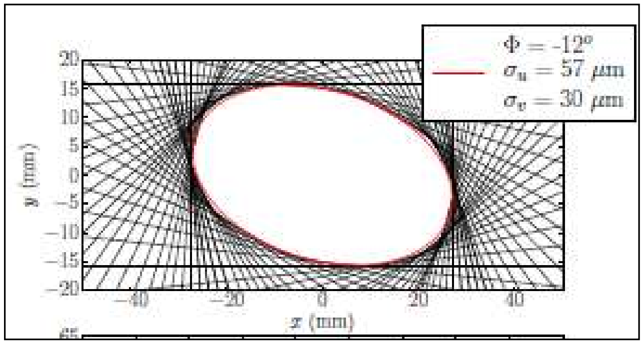
$$\sigma_x = \frac{\lambda L}{\pi D}\sqrt{\frac{1}{2}\ln\frac{1}{V}}$$

Double Aperture Interferometry: Rotating Mask

- SRI limitation: provide **only 1d measurement** → the 2-aperture orientation defines direction analysis
- Using a rotating mask, we can fully reconstruct beam profile by fitting σ for different rotation angles
- Multiple interferograms (at least 4) need to be done → 2D reconstruction cannot be done in real time

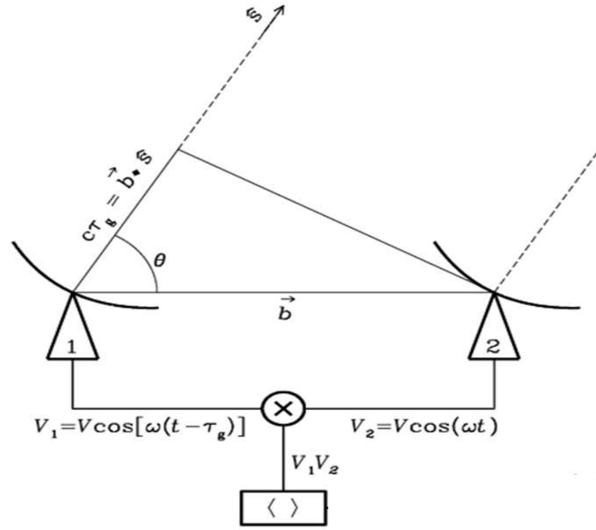




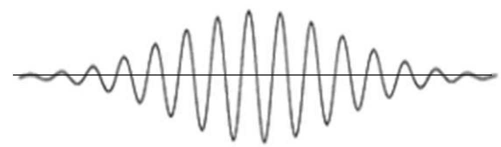


*L.Torino and U.Iriso, PRST-AB 19, 122801 (2016)

Influence from Radio-Astronomy



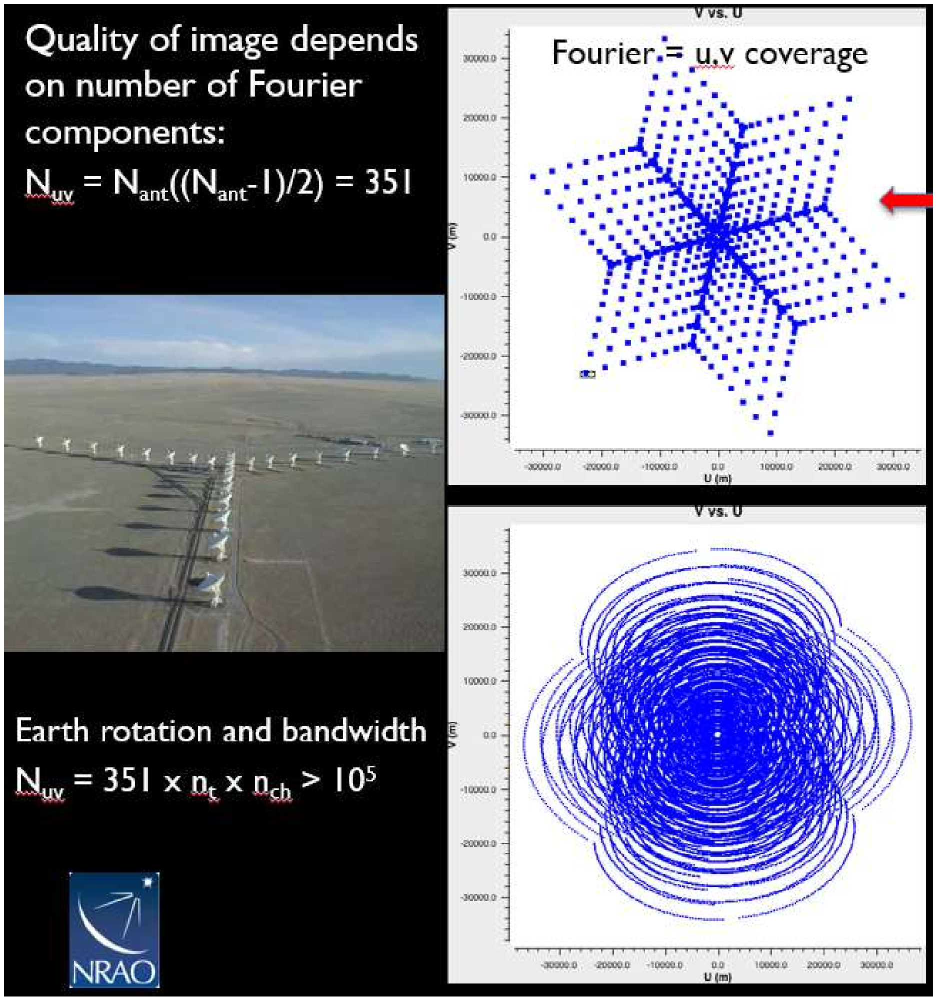
$V_{i,j} = E_i \times E_j^* = A_{i,j}, \phi_{i,j}$



- Coherent amplification of voltages, E
- Geometric delays set by electronics
- Images made from Fourier transform of visibilities

Antennas → apertures
Radio measurs. → visibilities

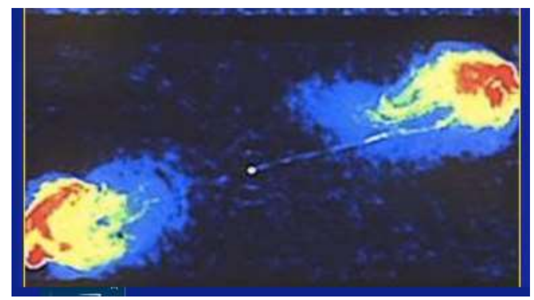
Influence from Radio-Astronomy



Apertures ← → Antennas (telescopes)

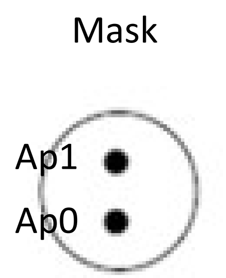
27 antennas at at the Very Large Array telescope (NRAO – New Mexico, USA)

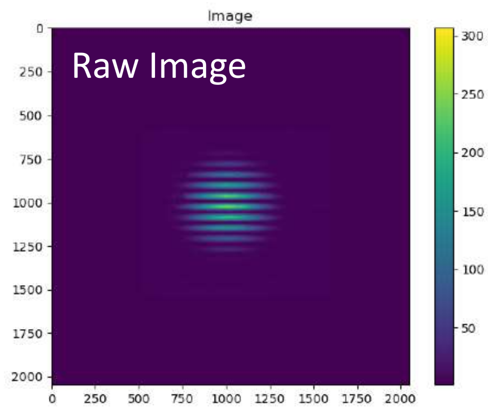
Able to image stars with this shape, why not a Gaussian beam??

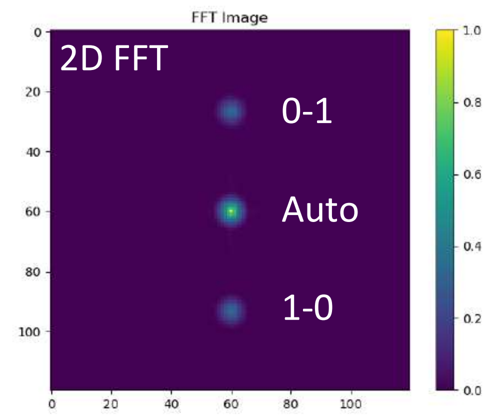


➔ For Synchrotron Radiation Interferometry, working in the **Fourier Domain** and working with **more apertures** will have advantages

Fourier Domain: 2-apertures







Fourier Domaim (u,v) image:

- Central Peak: Autocorrelation → $V_{00} \propto I_0 + I_1$
- Top peak: Visibility between apertures 0,1 → $V_{01} \propto 2\frac{\sqrt{I_0 I_1}}{I_0+I_1}\gamma_{01}$

- I_0, I_1 : Intensities of apertures 0,1.

V_{00} and V_{01} calculated from the (complex) Sum within the circle of the peak

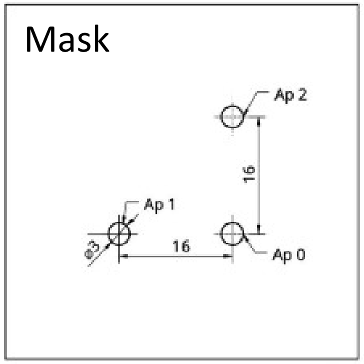
We could fit the two points with a normalized Gaussian in the Fourier (u,v) plane and get the coherence length σ_C:

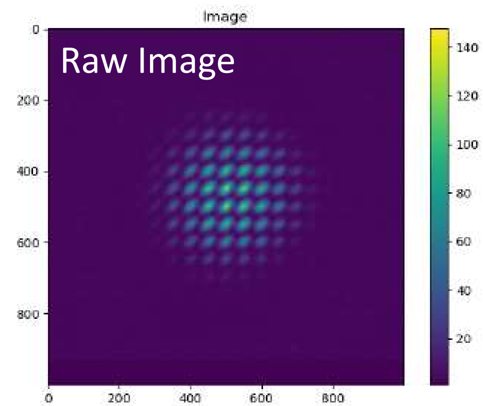
$$I(v) = \exp(-v^2/2\sigma_C^2)$$

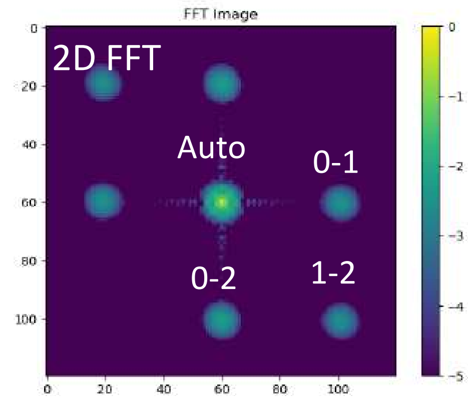
The beam size (σ) is inferred using VCZ theorem:

$$\sigma=\lambda\cdot L/(\pi\cdot\sigma_C)$$

Fourier Domain: 3-apertures







- Central Peak: Autocorrelation → $V_{00} \propto I_0 + I_1 + I_2$
- Peak 0,1: Visibility between apertures 0,1 → $V_{01} \propto 3\frac{\sqrt{I_0 I_1}}{I_0+I_1+I_2}\gamma_{01}$
- Peak 0,2: Visibility between apertures 0,2 → $V_{02} \propto 3\frac{\sqrt{I_0 I_2}}{I_0+I_1+I_2}\gamma_{02}$
- Peak 1,2: Visibility between apertures 1,2 → $V_{12} \propto 3\frac{\sqrt{I_1 I_2}}{I_0+I_1+I_2}\gamma_{12}$

V_{ij}, calculated from the (complex) Sum within the circle of the peak (i,j)!

Assuming a flat Gaussian beam, and knowing (I_0, I_1, I_2), or if $I_0=I_1=I_2$.

→ Fit the three points with a normalized 2D-Gaussian: (σ_{Cx} and σ_{Cy} as free parameter) in each direction and retrieve the 2D beam again using VCZ

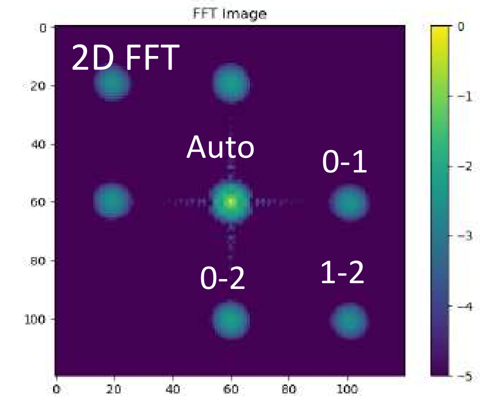
Fourier Domain: N-apertures

The process can be generalized as:

$$\|V_{ij}\| = \gamma_{\rm ij}\|G_i\|\|G_j\| \,,$$
$$\|V_{\rm auto}\| = \sum_i \|G_i\|^2 \,,$$

with $G_i = \sqrt{I_i}$

For a N-hole mask, → **$(N(N-1)/2)$ +1** distinct peaks in the Fourier Domain.



Summary:

- Known parameters: Vij (Visibilities) measured in Fourier Dom.
- Unknown parameters:
    - 2D tilted Gaussian: (σ_u, σ_v, ϕ) → 3 unknowns
    - Intensities: G_i , for i=1… N → N unknowns

P_k = (N(N-1)/2) + 1

P_u= N +3

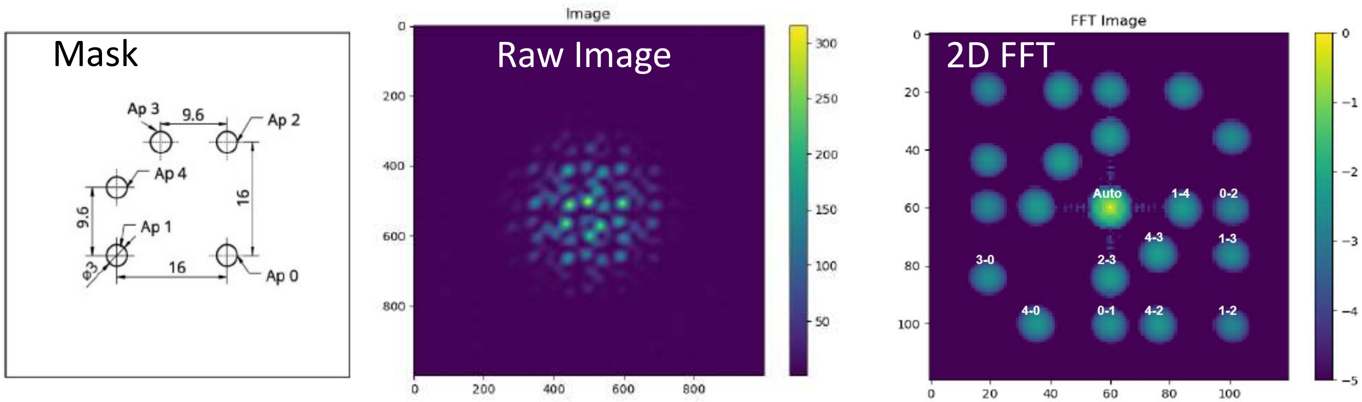
System fully characterized if Pk > Pu
So, for **5 holes**

Self-calibration Technique*:

- joint fit to the measured Vij to obtain both the Gaussian source parameters and hole gains (intensities)

*B. Nikolic, C. Carilli, U. Iriso, N. Thyagarajan, and L. Torino, "Two-dimensional synchrotron beam characterization from a single interferogram", PRAB, 27, 112802 – Published 25 November, 2024

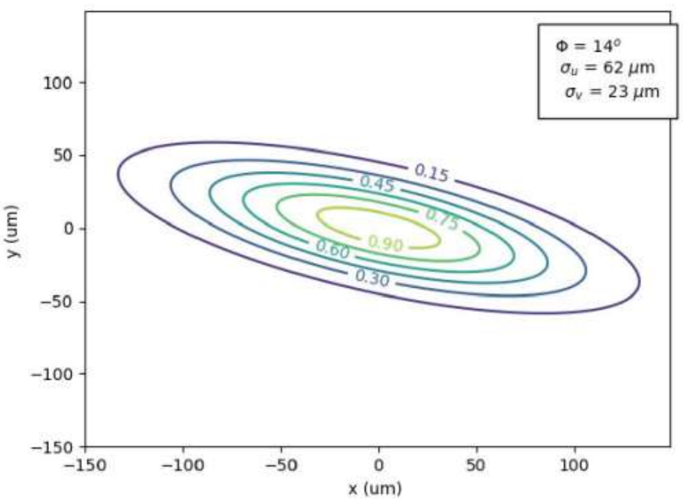
Results with 5-apertures



5 apertures

↓

11 distinct peaks



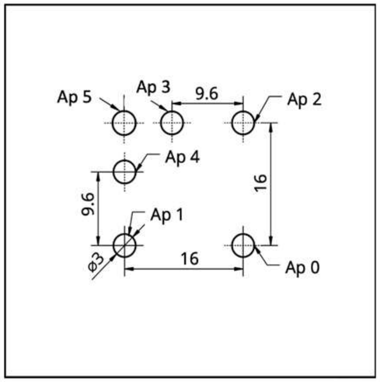
Electron Beam Reconstruction from a single interferogram.
Results in agreement with other methods

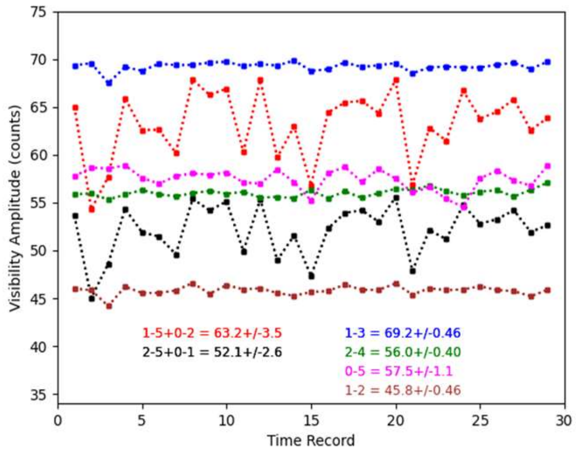
Method	Major (μm)	Minor (μm)	Tilt Angle
2ap (3 ms)	61.7 ± 1.5	25.5 ± 1.5	$16.6°$
5ap (1 ms)	59.6 ± 0.1	23.8 ± 0.5	$15.9° \pm 0.2°$
LOCO	57.5	20.6	$14.9°$
Pinhole	58.5	24.6	$14.9°$

*B. Nikolic, C. Carilli, U. Iriso, N. Thyagarajan, and L. Torino, “Two-dimensional synchrotron beam characterization from a single interferogram“, PRAB, 27, 112802 – Published 25 November, 2024

Why Non-Redundant Apertures?

→ To identify the peaks in the Fourier domain
→ Baselines with the same distance & direction produce blurring and decoherence in the Fourier space





Redundant combination:

- (1,5 – 0,2), Red
- (2,5 – 0,1), **Black**

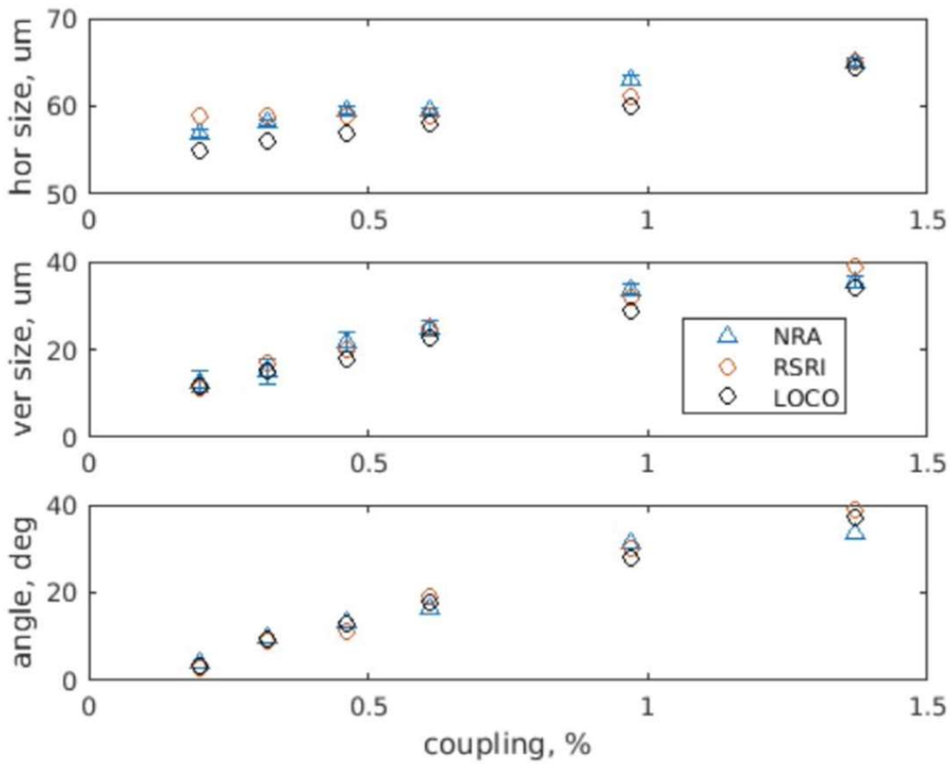
rms for redundant samples ~6X larger

Measurements while Varying Beam Coupling

- Vertical beam size scan changing the machine coupling with skew magnets.
- Results using the NRA mask (7-hole) agrees with:
  - Rotating (double aperture) SRI
  - LOCO Analysis

The minimum beam size we can achieve at ALBA (11um) is well measured.

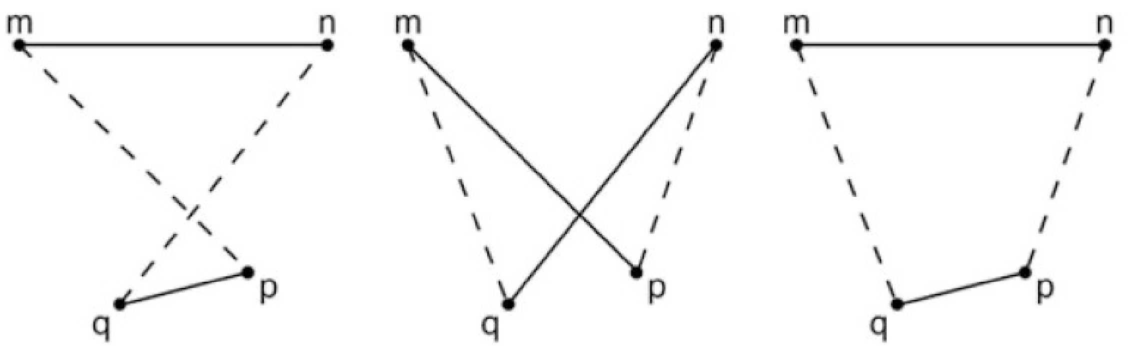
Currently doing SRW simulations to see the ultimate limit of this technique



U. Iriso, L. Torino, C. Carilli, B. Nikolic, N. Thyagarajn, New interferometric aperture masking technique for full transverse beam characterization, Proc. of IBIC24, 2024

Closure Amplitude Technique

- Alternative technique to solve the interferogram
- Closure amplitude: closed loop of 4-aperture elements, or special data combination on 4-elements (quad)



$$\frac{|\mathcal{V}_{mn}||\mathcal{V}_{pq}|}{|\mathcal{V}_{mp}||\mathcal{V}_{nq}|}$$

>>Twiss et al. (1960)
>>Thomson, Moran, Swenson (2017)

- Closure amplitude is independent of antenna-based gain corruptions (hole illumination irregularities)
- Requires a closed loop of >= 4 elements
- N(N-3)/2 independent closure amplitudes

- Played a key role in astronomical discoveries (super-massive black holes in M87 and Milky Way galaxies).

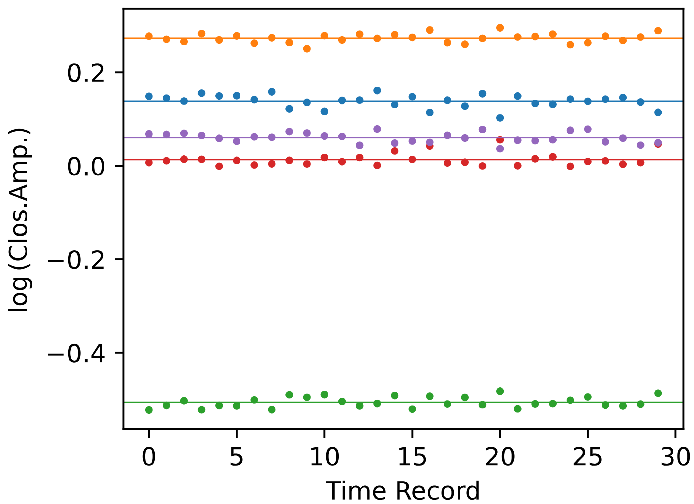
Closure Amplitude Technique

Beam shape estimated directly from closure amplitudes from uncalibrated visibilities

$$|V_{pq}(t)| = \gamma(\mathbf{u}_{pq})\,|G_p(t)|\,|G_q(t)|\,.$$

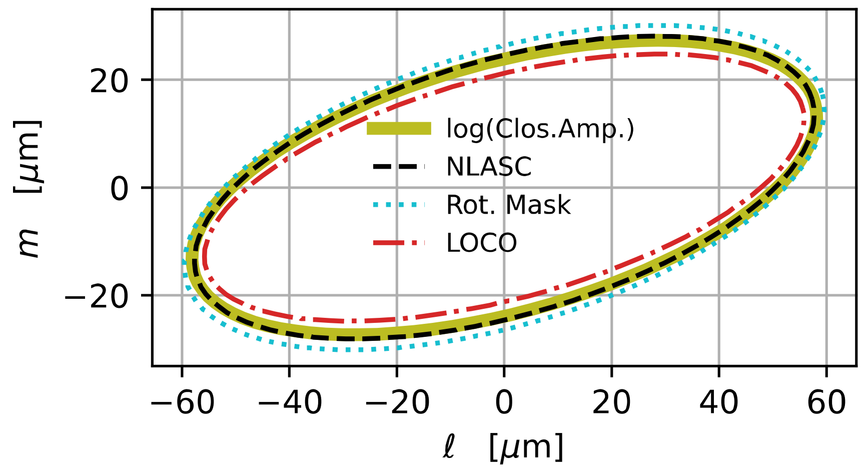
$$A_{pqrs} = \frac{|V_{pq}(t)|\,|V_{rs}(t)|}{|V_{ps}(t)|\,|V_{rq}(t)|} = \frac{|\gamma(\mathbf{u}_{pq})|\,|\gamma(\mathbf{u}_{rs})|}{|\gamma(\mathbf{u}_{ps})|\,|\gamma(\mathbf{u}_{rq})|}$$

A_{pqrs} is independent of G!



Closure amplitudes measured from uncalibrated visibilities are stable in time

$$\log A_{pqrs} = \frac{1}{2}[(u_{ps}^2 + u_{rq}^2 - u_{pq}^2 - u_{rs}^2)\,a + (u_{ps}v_{ps} + u_{rq}v_{rq} - u_{pq}v_{pq} - u_{rs}v_{rs})\,b + (v_{ps}^2 + v_{rq}^2 - v_{pq}^2 - v_{rs}^2)\,c]\,.$$



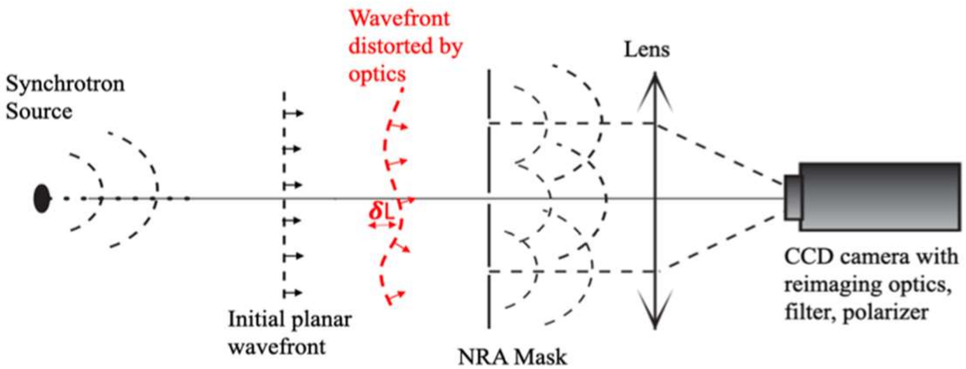
3 beam shape parameters (a, b, c) to be estimated from 5 closure amplitude (quad) measurements

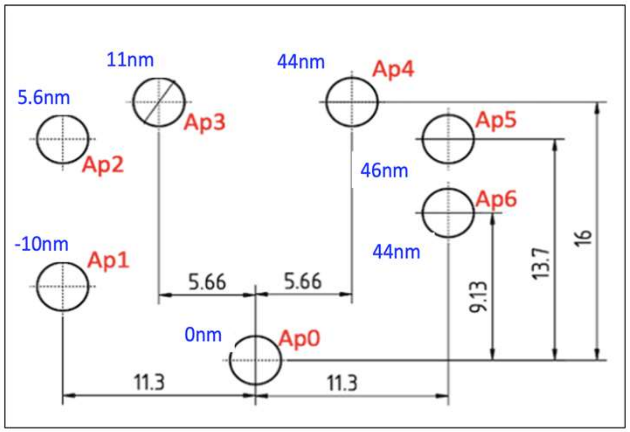
Thyagarajan et al., 2025, https://opg.optica.org/josaa/abstract.cfm?uri=josaa-42-9-1261

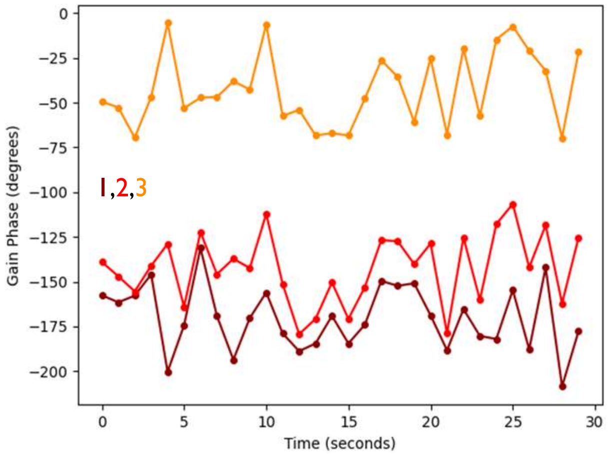
- Introduction: 2-apertures Interferometry
- Fourier Domain: from 2-holes to N-holes Interferometry
- Non-Redundant Aperture Interferometry: Self-Calibration
- Closure Amplitude Technique
- Other Applications:
  - WaveFront Sensing
  - Arbitrary beam shapes – halo meas
  - From ALBA to James Webb Telescope
- Conclusions

WaveFront Sensor

Gain Phase solutions encode **δ**L , so it can be used as WaveFront Sensor



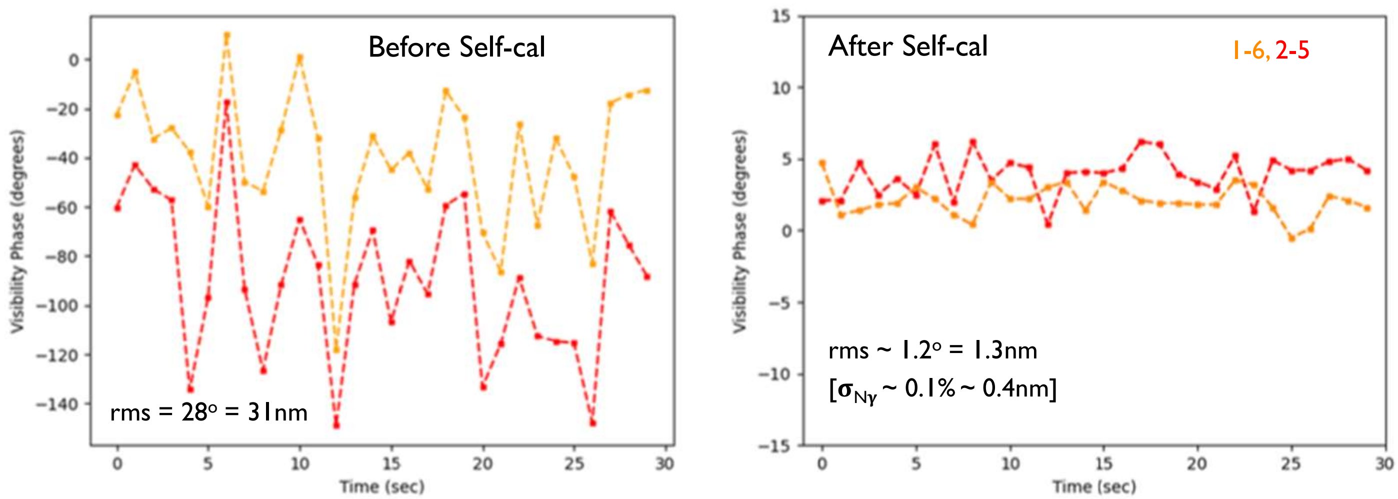




$\delta L = \lambda \times \theta_i / 360^\circ$

Variations ~ 20º (6nm) are correlated between neighboring holes

WaveFront Sensor



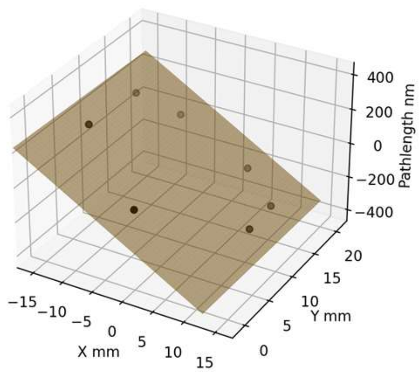
- Mean = static pathlength differences ('metrology')
- Variations (millisec) = vibrations and turbulence
  - **δΦ** is correlated between pairs-of-holes => systematic **δ**L, not noise
  - rms after self-cal much lower => measuring frame **δ**L to high significance

WaveFront Sensor

Mirror Rotation Tests

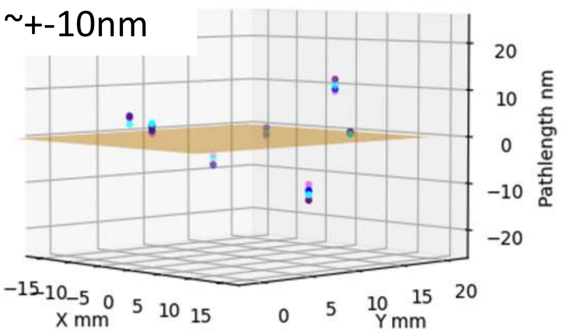
Wavefront Sensor “calibrated” using a mirror whose rotation can be controlled down to 0.1urad
Move the mirror by a known amount, and measure the wavefront distortion from the gain phases

2” input tilt
2.15” fit tilt



- Planar fit to the 2” tilt data
- Tilts up to +/-50 nm across mask (~5urad); random frame-to-frame
- Mean (static) residuals after subtracting the best fit plane to all the tilted mirror experiments show deviations from plane ~ +/-10nm
- Non-plannar distortions are reproduced between experiments within +/-1nm

Static residuals average: ~+-10nm



APPLICATION:

- Control of possible mirror deformation due to heating, long-term contamination, etc

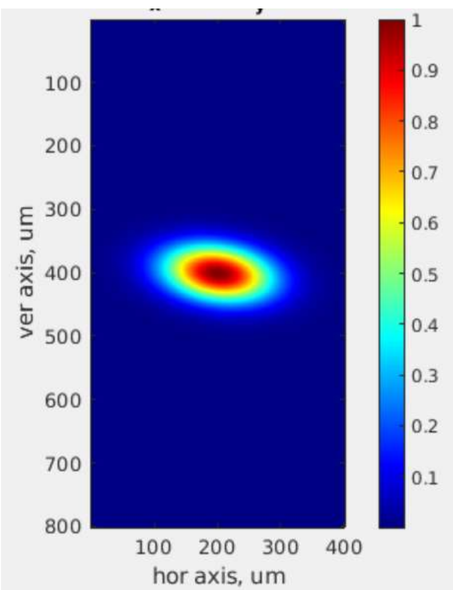
IBIC 2025, TUPMO09 https://meow.elettra.eu/90/pdf/TUPMO09.pdf

Imaging Double-Gaussian Beams

Artificially create beams with different shapes → re-construction using NRA?

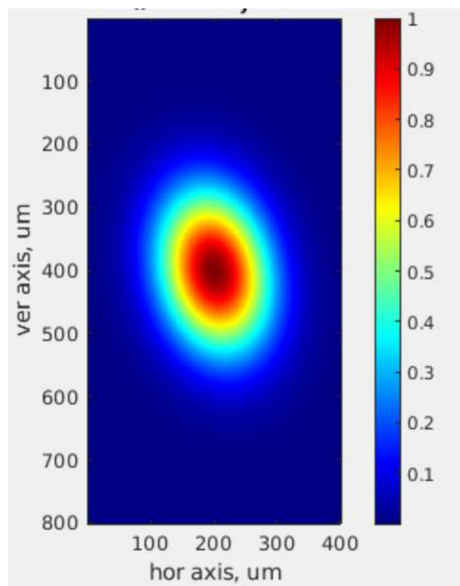
Using the Bunch-by-bunch system, we mix different vertical beam sizes within the filling pattern

A. Beam stable (as in Operation):
(σ_x, σ_y) = (60, 30)um; $\phi = 8º$



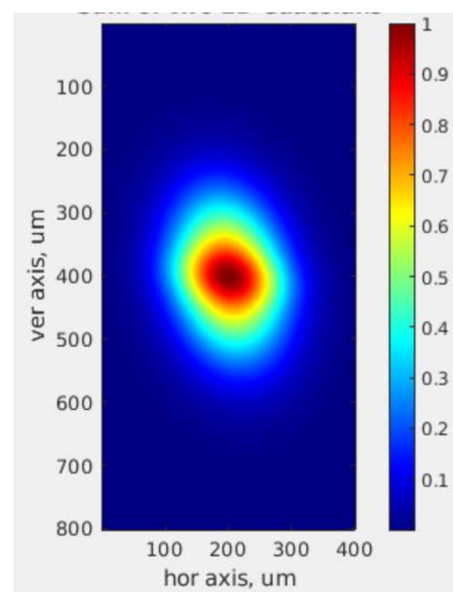
Pinhole Image

B. Coherent Excitation (Qv+Qs):
(σ_x, σ_y) = (60, 90)um; $\phi = 12º$



Pinhole Image

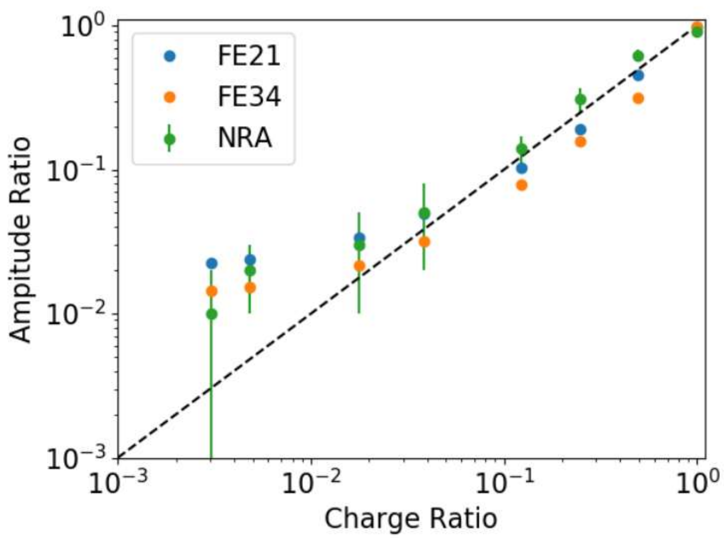
(100 bchs of “A” + 340 bchs of “B”):
Beamshape characterization?



Pinhole Image

Imaging Double-Gaussian Beams

- Fit a sum of two Gaussians, with the constraint that their sum of amplitudes is 1
- Dual purpose experiment:
  - Reproducing non-Gaussian beam shapes
  - Possible Halo Measurements: reduction of excited bunches until < 1/440



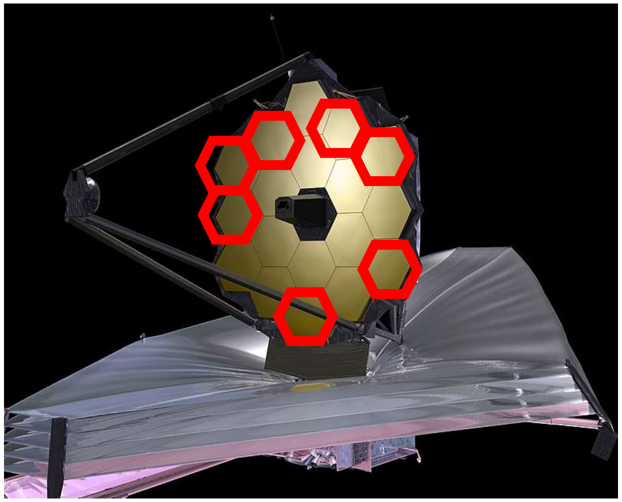
- Amplitude of the two Gaussians (Excited and Non-Excited beams) is compared with the charge ration measured using the Photon Counting.
- The beam shape is well reproduced down to ~1% (similar precision obtained with pinhole cameras FE21 & FE34)

- The complexity of the source that can be derived will depend on the completeness of the u,v sampling.
- Mask design should be optimized to reproduce different beam shapes

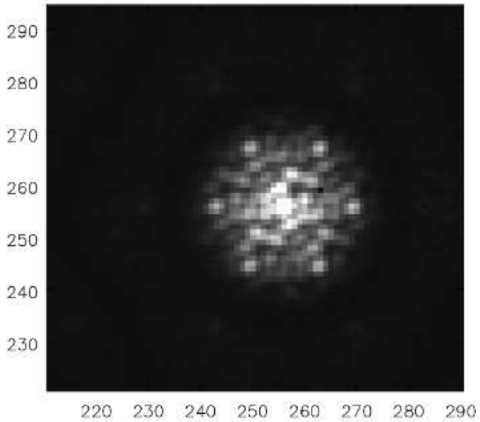
IBIC 2025, TUPMO09 https://meow.elettra.eu/90/pdf/TUPMO09.pdf

From ALBA to the James Webb Telescope

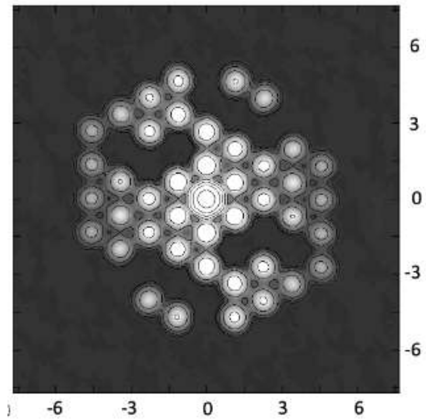
- JWST can have up to 18 mirrors (apertures), but one can play with them to produce Aperture Mask Interferometry (AMI).
- Self-calibration NRA remains a key capability for imaging at the diffraction limit of the JWST, in particular, for close star-planet systems.
- This methodology was long used in radio astronomy, but not, to date, in optical interferometry*.



Interf. NRA with 7-mirrors



Fourier Transform.



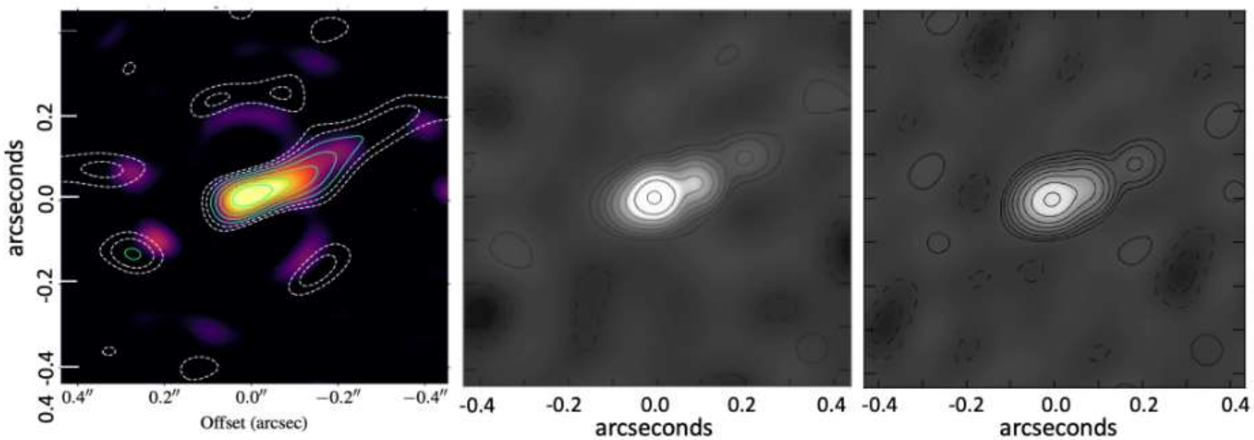
*C. Carilli, “A New Method for Aperture Masking Interferometric Imaging: Demonstration with the JWST”, Optical Engineering SPIE, 2025

From ALBA to the James Webb Telescope

- Reliable double-star system image reproduction using self-calibration techniques
- Explore other aspects ofthe JWST system → 'real-time' wavefront sensor

Imaging of dusty binary star WR137

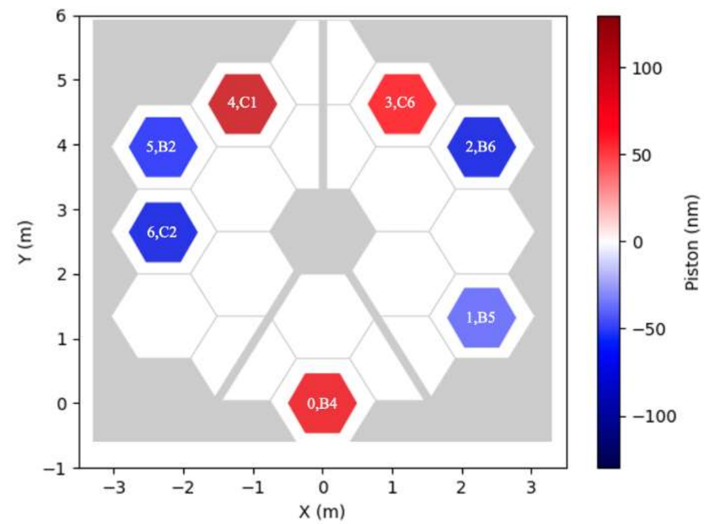
'Real-time'wavefront error sensor.



'standard' method
(R. Lau et al**)

NRA interf., λ=4.8um
(C. Carilli et al*)

NRA interf., λ=3.8um
(C. Carilli et al*)



*C. Carilli et al, "A New Method for Aperture Masking Interferometric Imaging: Demonstration with the JWST" Optical Engineering SPIE, 2025

** R. Lau, et al., "A first look with JWST aperture masking interferometry: dust around the wolf–rayet binary WR 137 beyond the rayleigh limit," The Astrophysical Journal 963, 127 (2024).

Conclusions

- New Technique for Full 2D transverse beam characterization from a single interferogram, produced by a multi-hole Non-Redundant Aperture mask.
  - Accounts for different aperture illumination (self-calibration)
  - Can also be solved using the "Closure Amplitudes" technique

- Advantages wrt "classical" double-aperture :
  - Full 2D transverse beam reconstruction in a single interferogram
  - More holes → more light → better SNR, better resolution…

- Other benefits:
  - Wavefront distortions can be obtained using the Gain Phases: detected tilts ~urad scale
  - Double-Gaussian shapes can also be reproduced with the NRA
  - SNR (or Dynamic range) of a signal can be increased by optimizing the mask design: masks can be optimized for possible Halo Measurements (work in progress)

- Multi-disciplinary collaboration between Astronomers and Accelerator Physicists pays off!

Publications

https://indico.synchrotron-soleil.fr/event/76/, New interferometric aperture masking technique for full transverse beam characterization using synchrotron radiation, **DEELS24**, June 2024

https://arxiv.org/abs/2406.02114 , Deriving the size and shape of the ALBA electron beam with optical synchrotron radiation interferometry using aperture masks: technical choices, June 2024

https://opg.optica.org/josaa/abstract.cfm?uri=josaa-41-8-1513 , Laboratory demonstration of image plane self-calibration in interferometry, , **Journal of the Optical Society of America** , Aug. 2024

https://accelconf.web.cern.ch/ibic2024/doi/jacow-ibic2024-tup56/ , New interferometric aperture masking technique for full transverse beam characterization, **IBIC24**, Sept. 2024

https://journals.aps.org/prab/abstract/10.1103/PhysRevAccelBeams.27.112802 , Two-dimensional synchrotron beam characterization from a single interferogram, **Phys. Rev. Accelerator & Beams**, Nov. 2024.

https://opg.optica.org/josaa/abstract.cfm?uri=josaa-42-9-1261, Two-dimensional light beam shape characterization using interferometric closure amplitudes, **Journal of the Optical Society of America**.

https://arxiv.org/abs/2503.10820 , Self-calibration in two dimensional aperture mask optical interferometry: a new method for wavefront sensing, March 2025

https://meow.elettra.eu/90/pdf/TUPMO09.pdf , Exploiting NRA Interferometry as a Diagnostics Tool for Synchrotron Light Characterization, **IBIC25** (2025)

http://dx.doi.org/10.1117/12.3062598 , A new method for wavefront sensing using optical masking interferometry. **Proc. SPIE Conference 13619**, (18 September 2025);

https://arxiv.org/abs/2510.13502 , A New Method for Aperture Masking Interferometric Imaging: Demonstration with the JWST, **submitted to Optical Engineering (SPIE).** Oct. 2025

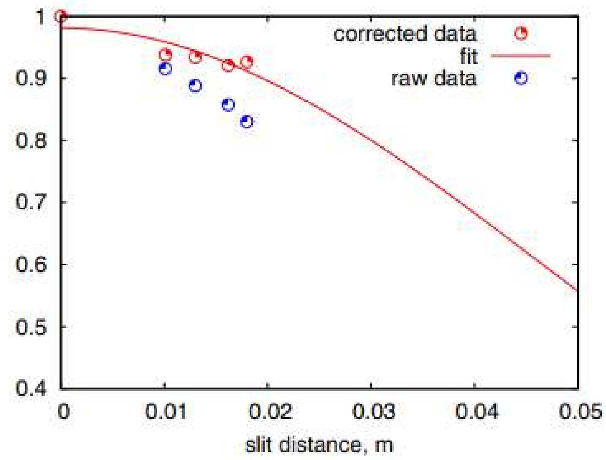
- US patent 12,104,901
- US patent app. No. 18/878,488
- US patent app. No. 19/079,876

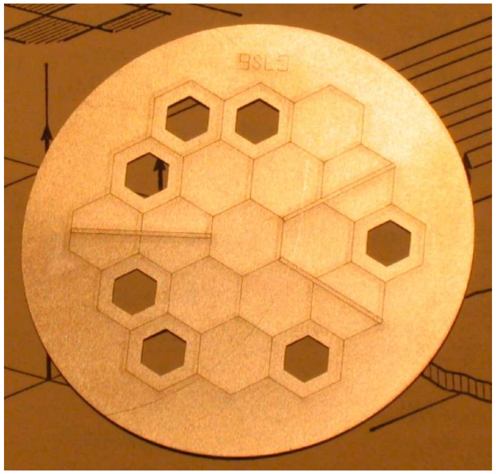
Extra

Double Aperture Interferometry: Intensity Imbalance

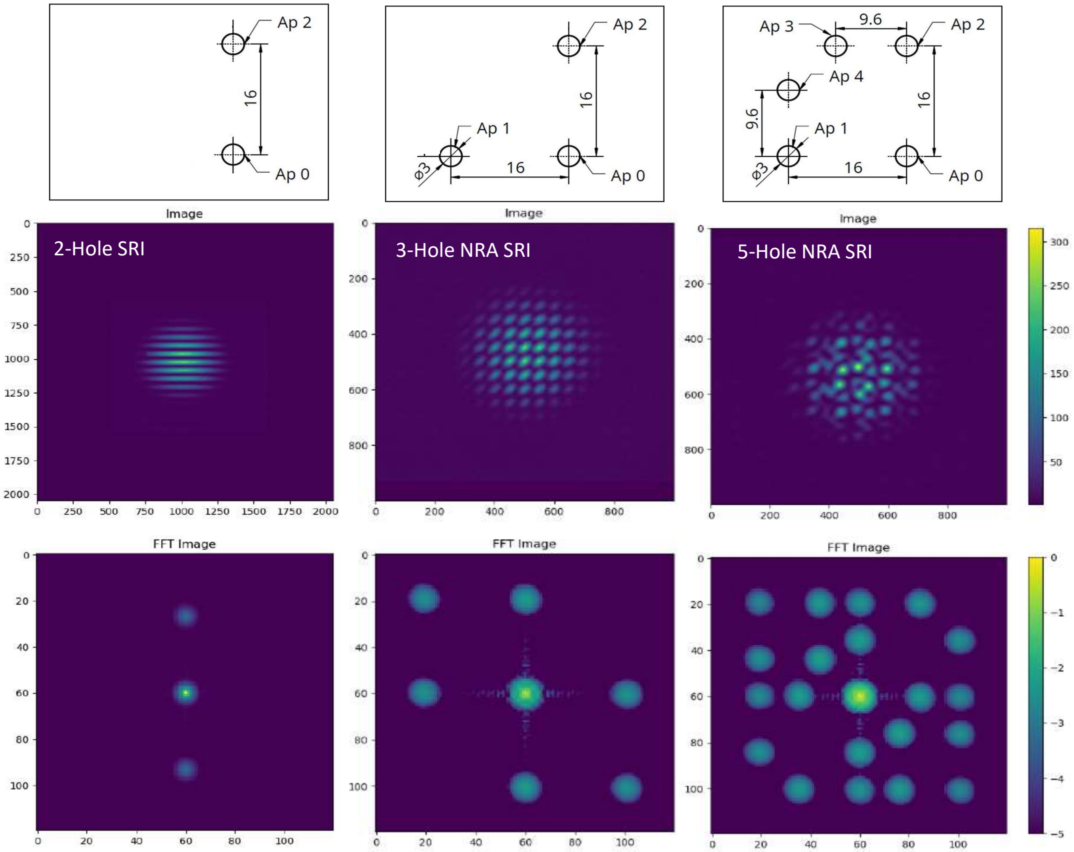
The Visibility V is related to the complex degree of coherence γ by a factor involving the intensities going through each hole I_1 and I_2 .

$$V = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2}|\gamma|.$$





James Webb Telescope Mask for 7-hole NRA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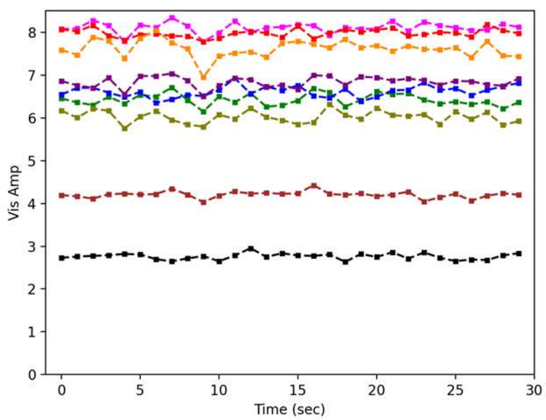
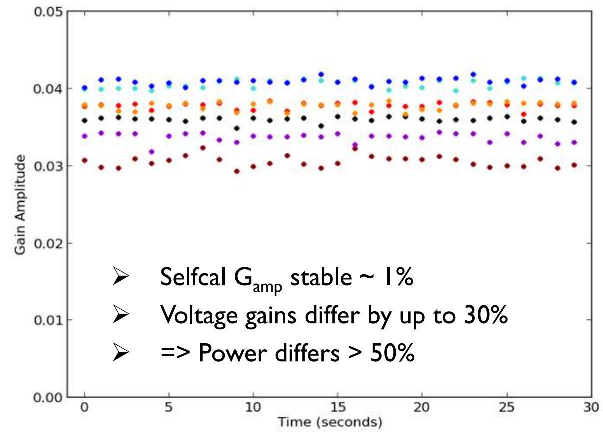
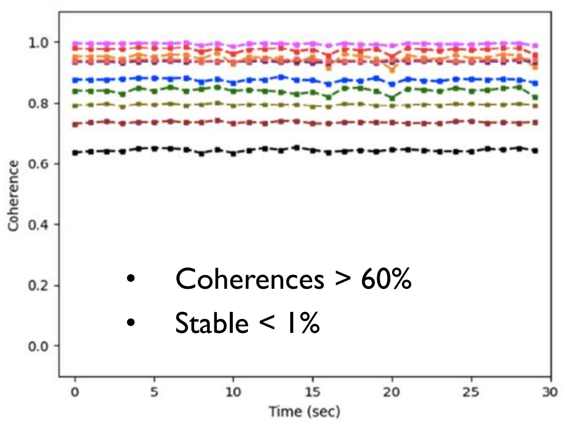
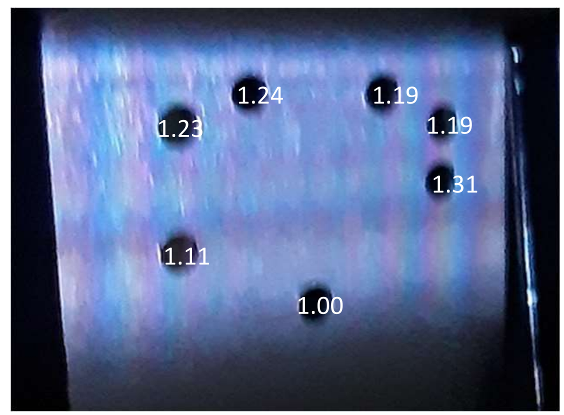
Ap 0
Image
2-Hole SRI
3-Hole NRA SRI
5-Hole NRA SRI
FFT Image

Self-calibration Amplitudes = Illumination





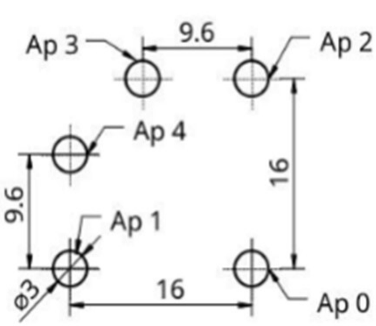




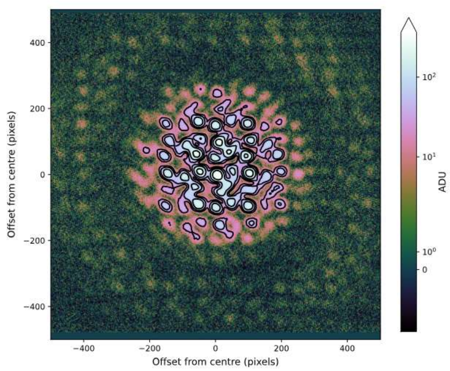
Alternative Techniques for σ<5um: NRA Interferometry

- Non-Redundant Aperture Interferometry using the Visible Light from FE01
- Direct measurement of the 2D coherence lenght of the photon beam with one single interferogram
- Extract beam sizes (and tilt angle) of the e-beam

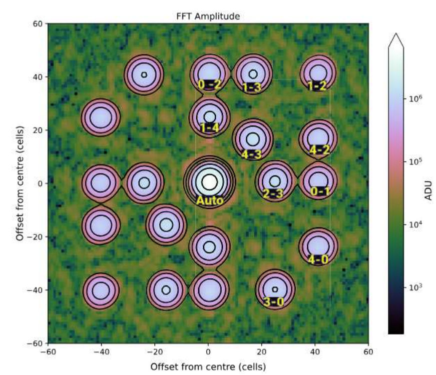
NRA Mask:



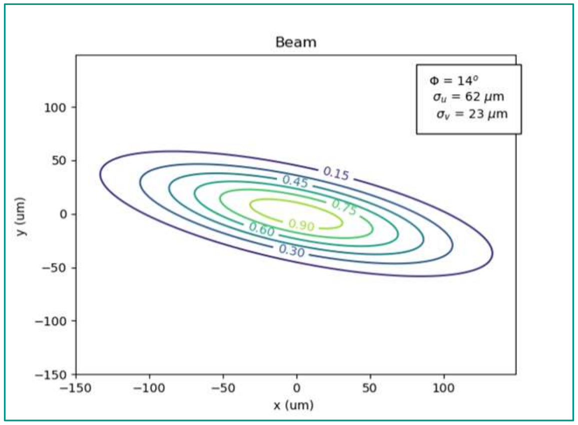
Direct Interf. Image



FFT Analysis



Electron Beam Image Re-construction



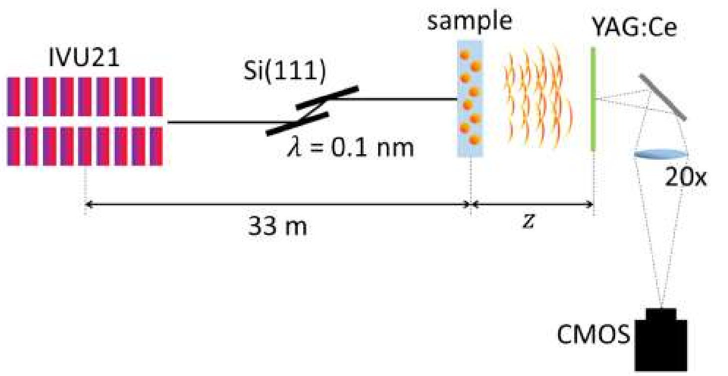
*New interferometric aperture masking technique for full transverse beam characterization, L. Torino et al, DEELS'24, https://indico.synchrotron-soleil.fr/event/76/ – Accepted for publication at PRAB-2024

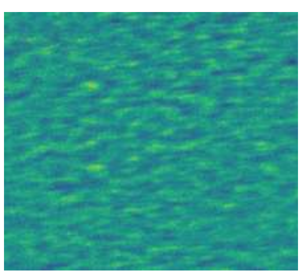
Alternative Techniques for σ<5um: X-ray HNFS

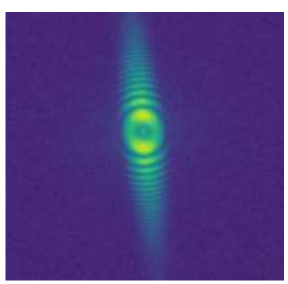
Two-dimensional electron beam size measurements with x-ray heterodyne near field speckles

M. Siano, B. Paroli, M. A. C. Potenza, L. Teruzzi, U. Iriso, A. A. Nosych, E. Solano, L. Torino, D. Butti, A. Goetz, T. Lefevre, S. Mazzoni, and G. Trad

Phys. Rev. Accel. Beams **25**, 052801 – Published 9 May 2022







Beam size measurements at NCD down to **σ = 4.5um**